Doppler Time-of-Flight Rendering

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Doppler Time-of-Flight (D-ToF) Imaging [Heide 2015]

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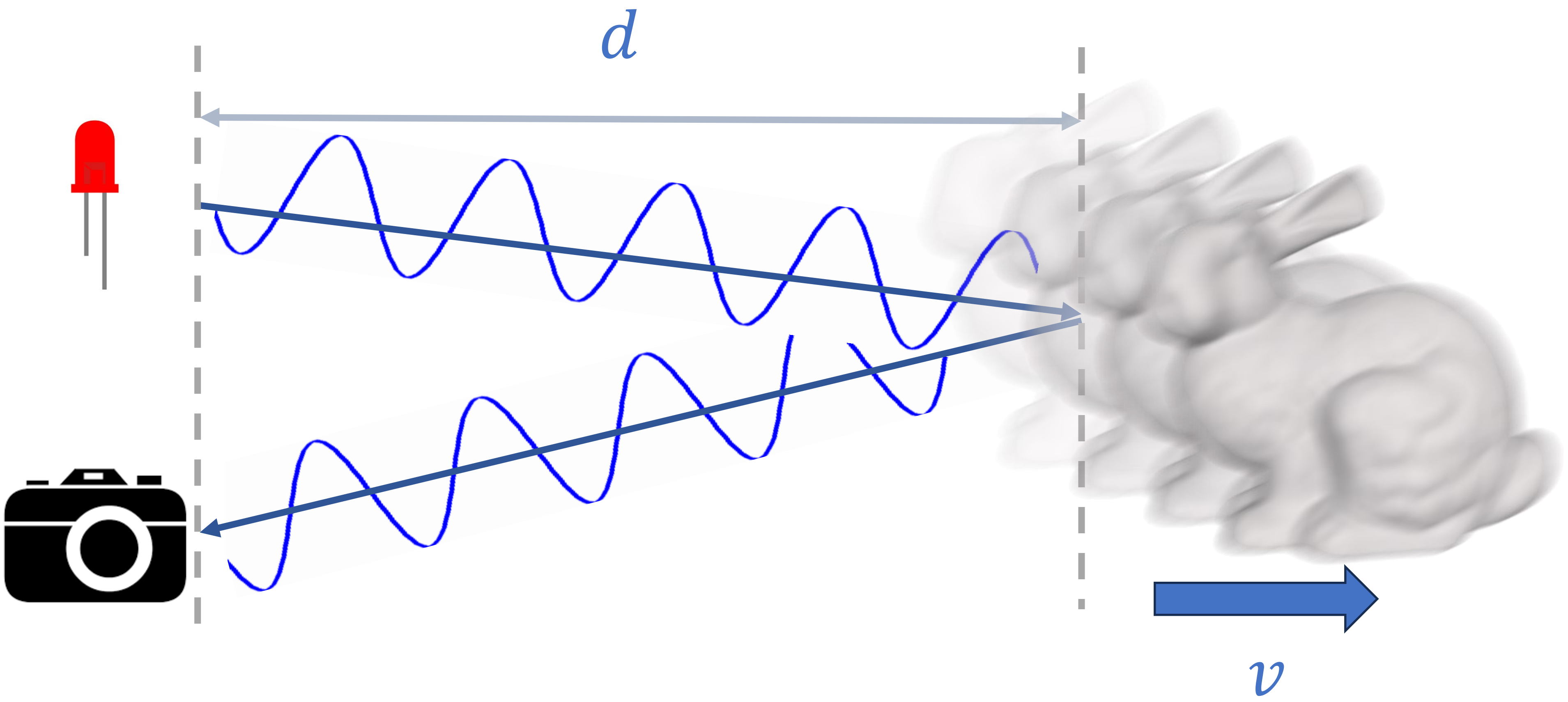
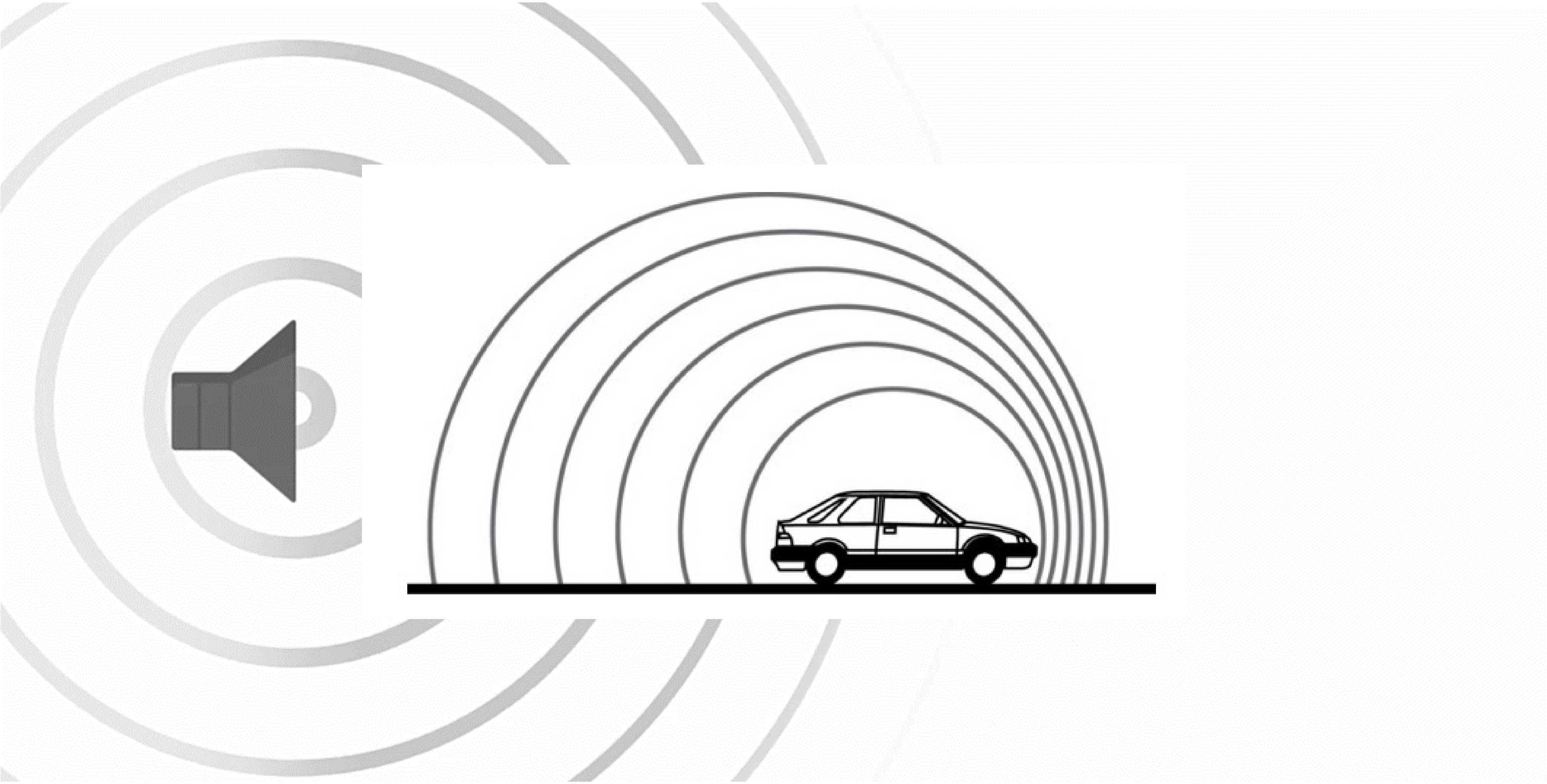


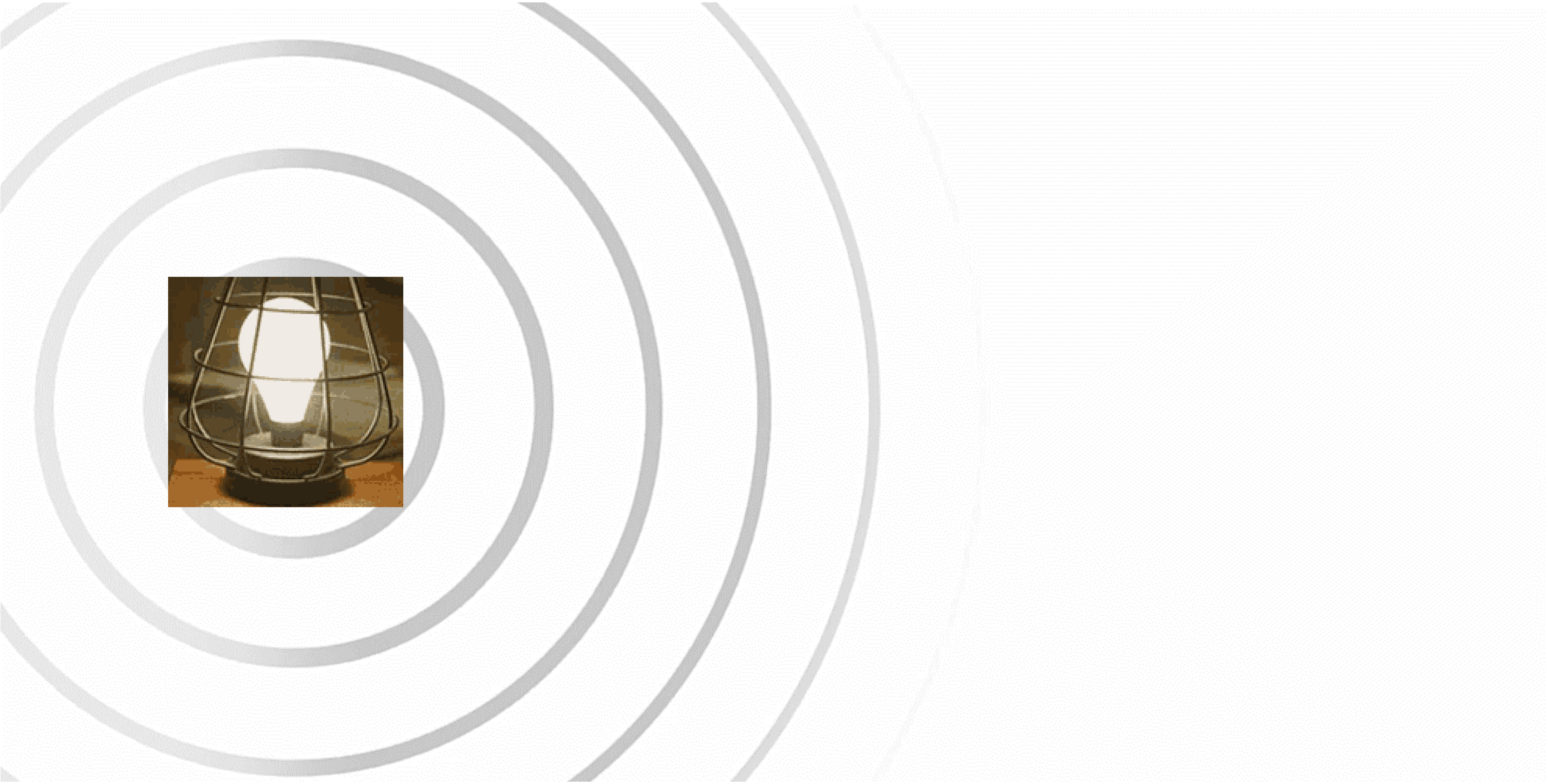
$d \propto$ Time-of-Flight

$v \propto$ Doppler Effect!

What is Doppler Effect?



Doppler Effect on Amplitude Modulation



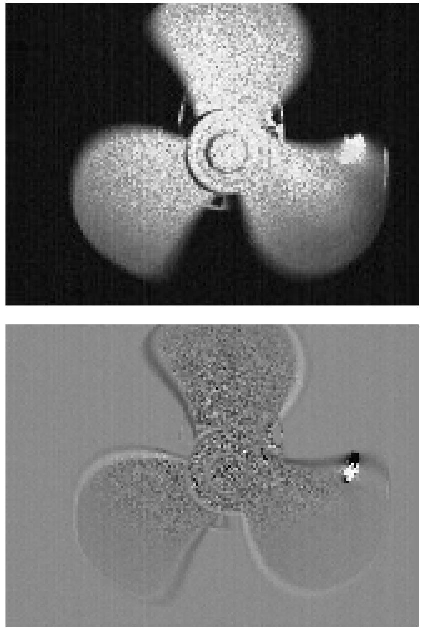
Doppler Time-of-Flight Imaging [Heide 2015]

Amplitude Modulated!

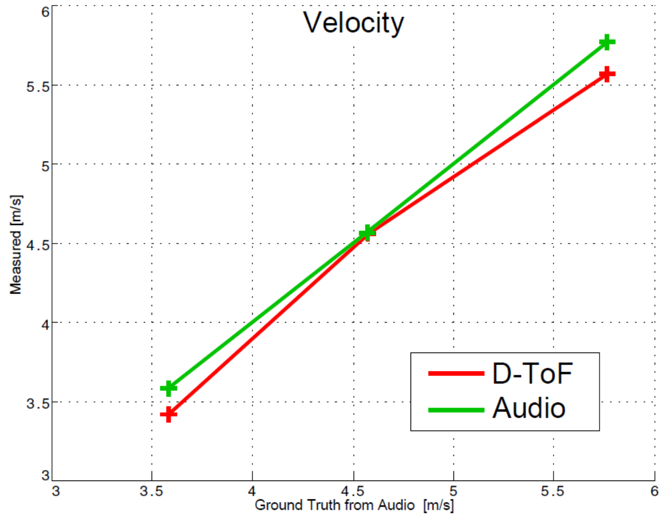
Light Source

Camera

Imaging system setup for D-ToF camera



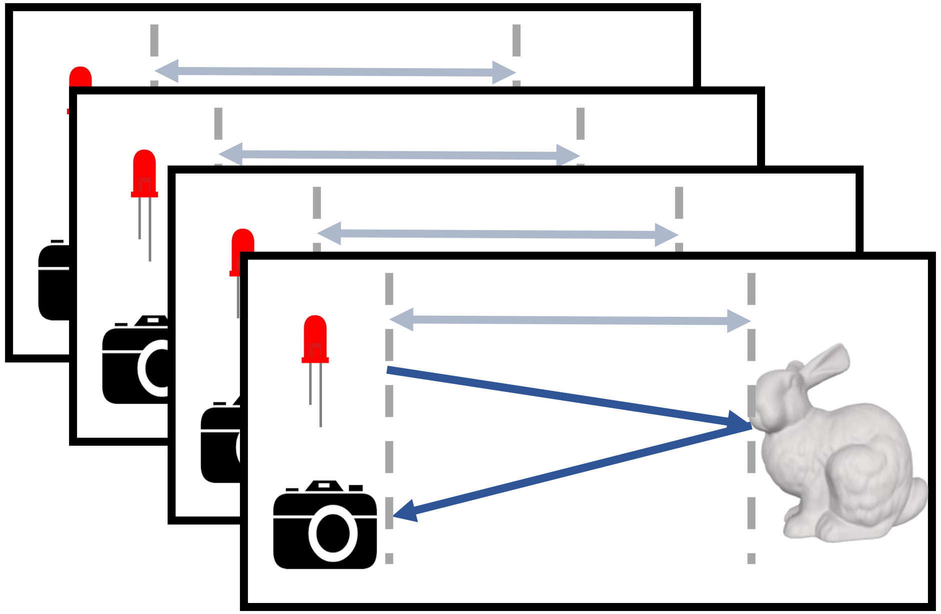
D-ToF measurements



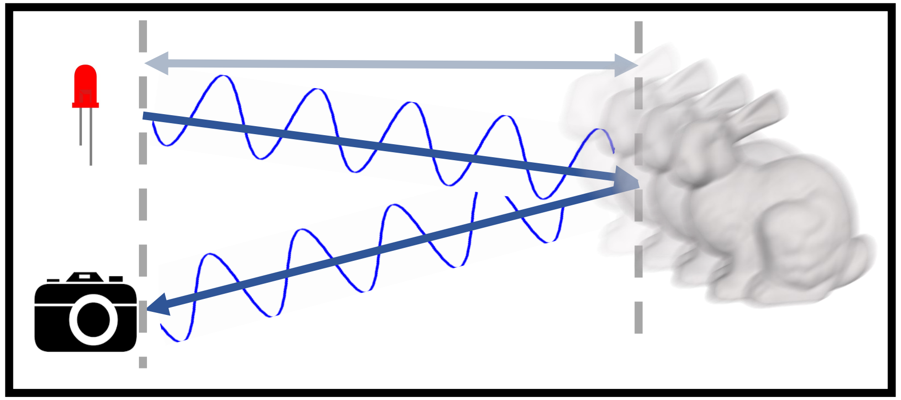
Radial Velocity Estimation

All images are from [Heide 2015]

Doppler Time-of-Flight Imaging : Advantage



Inter-frame method



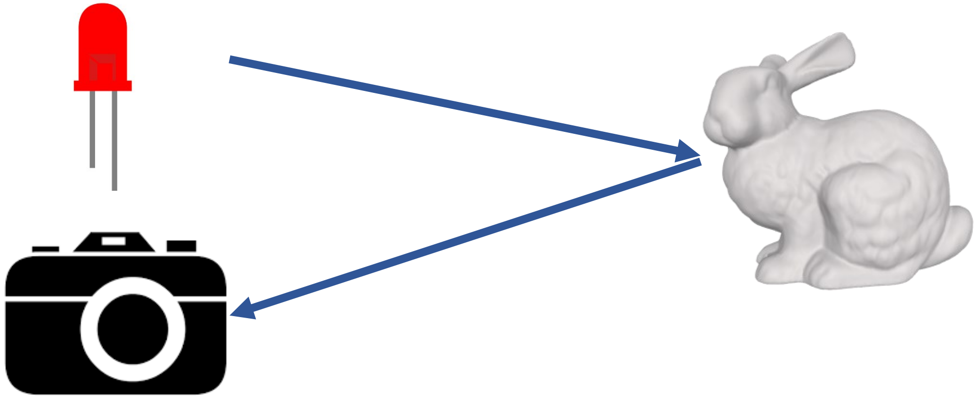
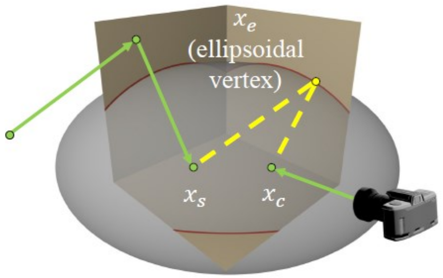
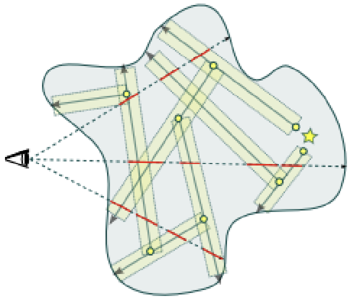
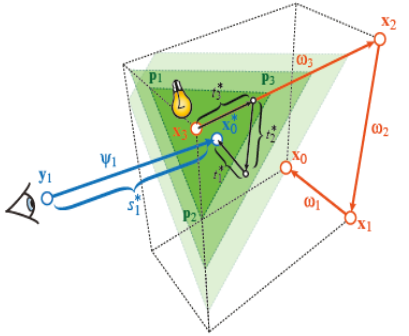
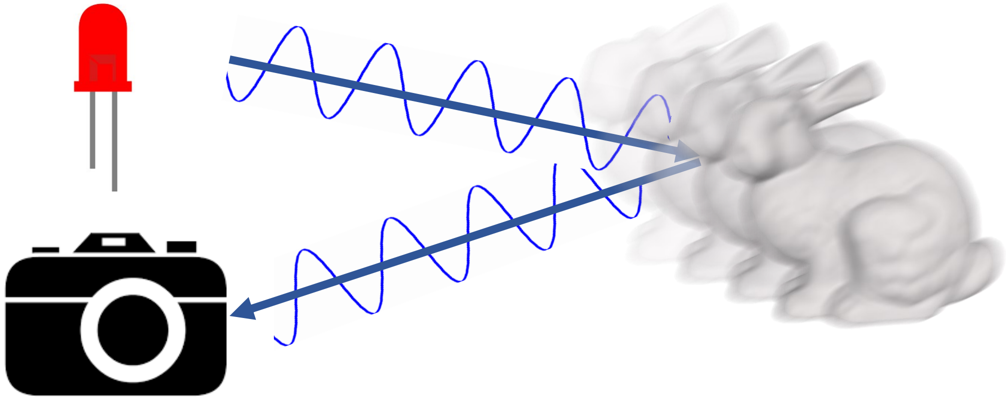
D-ToF camera

- × Multi-frame sensing
- × Long time interval

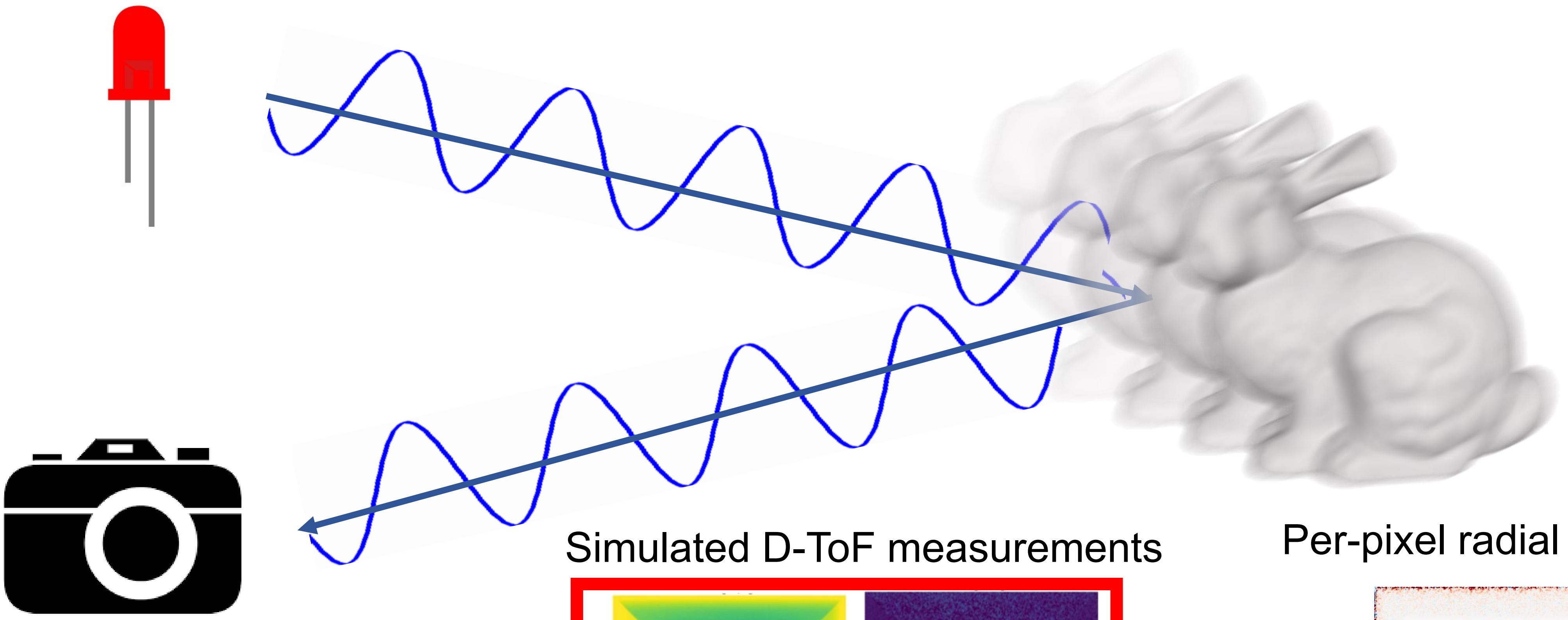
- ✓ Instant sensing
- ✓ Good for high-speed applications

Digital Twin for D-ToF Imaging System

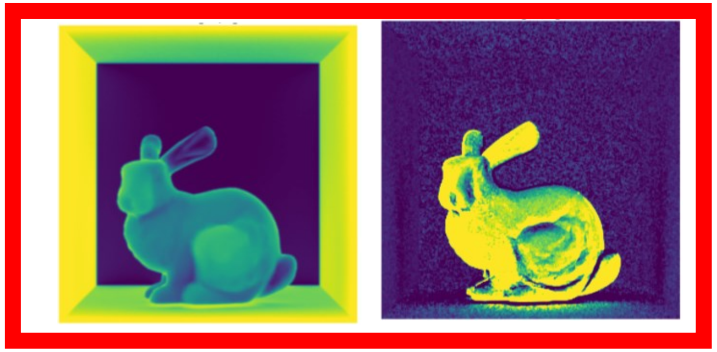
Digital Twin

	Imaging / Hardware	Rendering / Software
ToF		 [Pediredla 2019]  [Marco 2017, 2019]  [Liu 2022]
D-ToF		?

Doppler Time-of-Flight Rendering

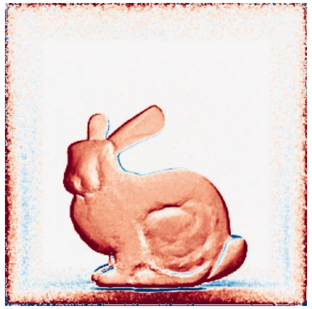


Simulated D-ToF measurements

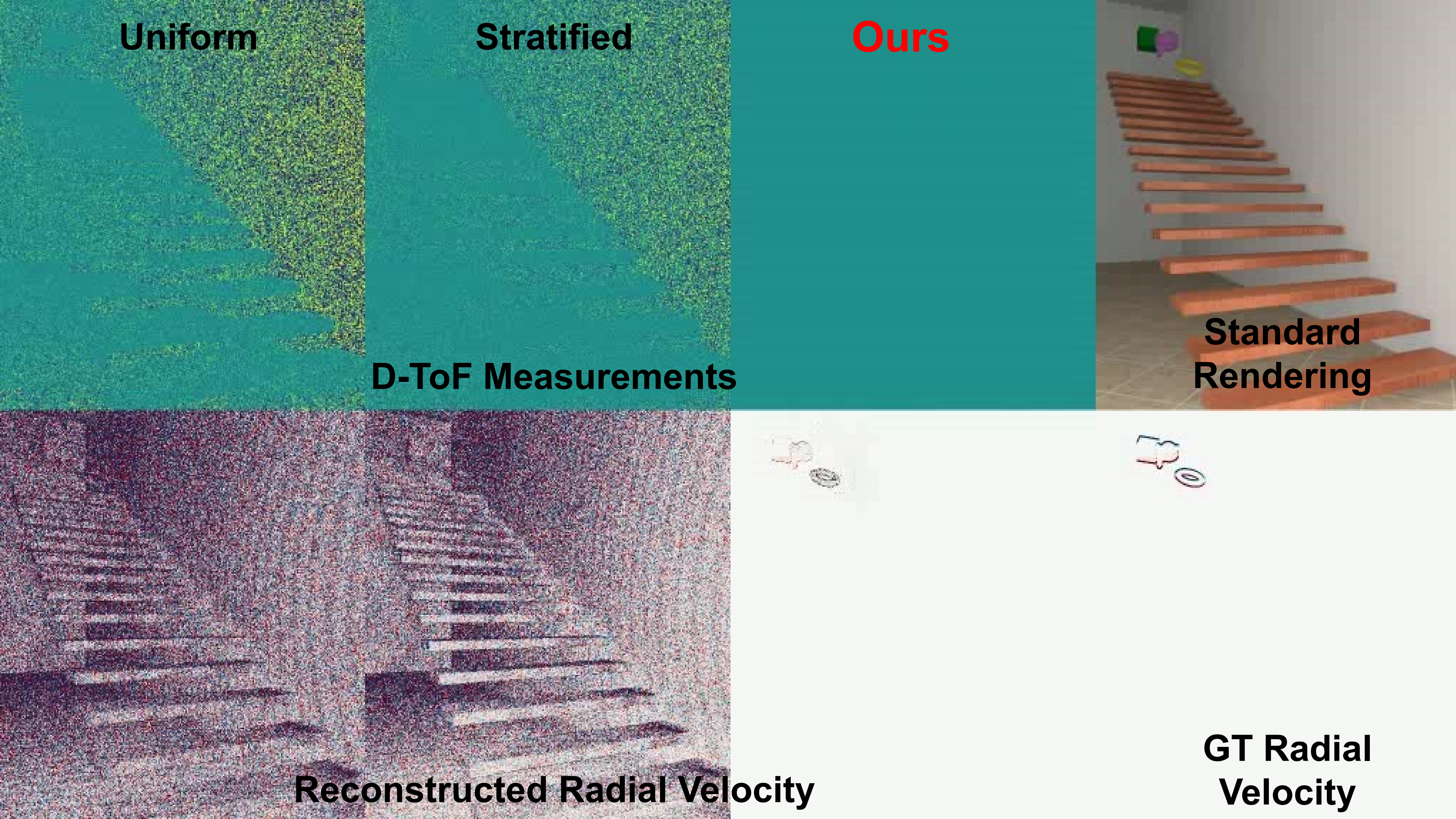


✓ **Physically Accurate**

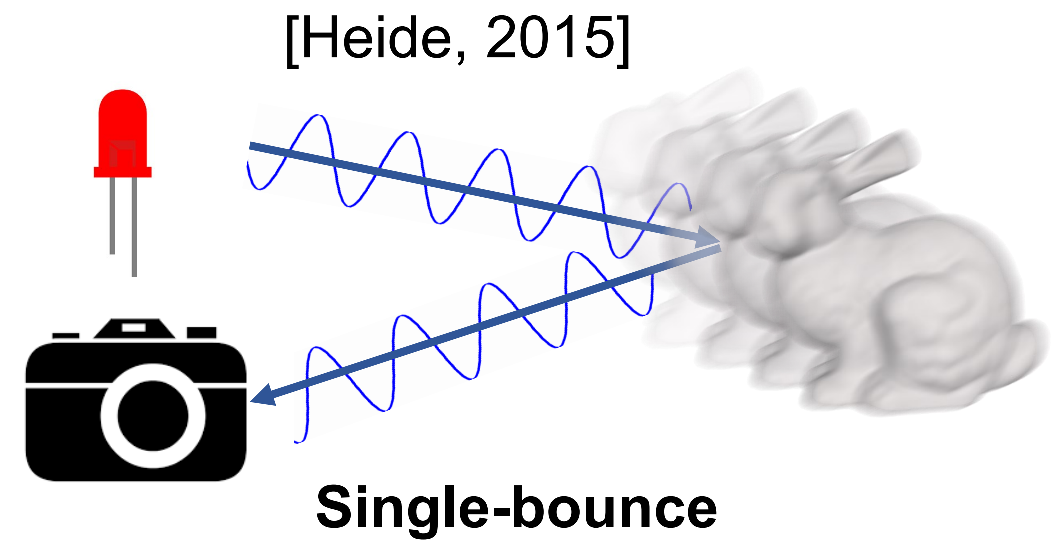
Per-pixel radial velocity

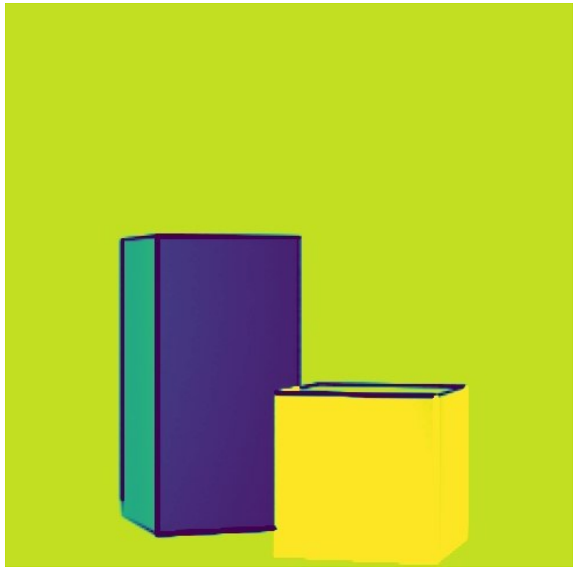


Red/Blue :
radial velocity

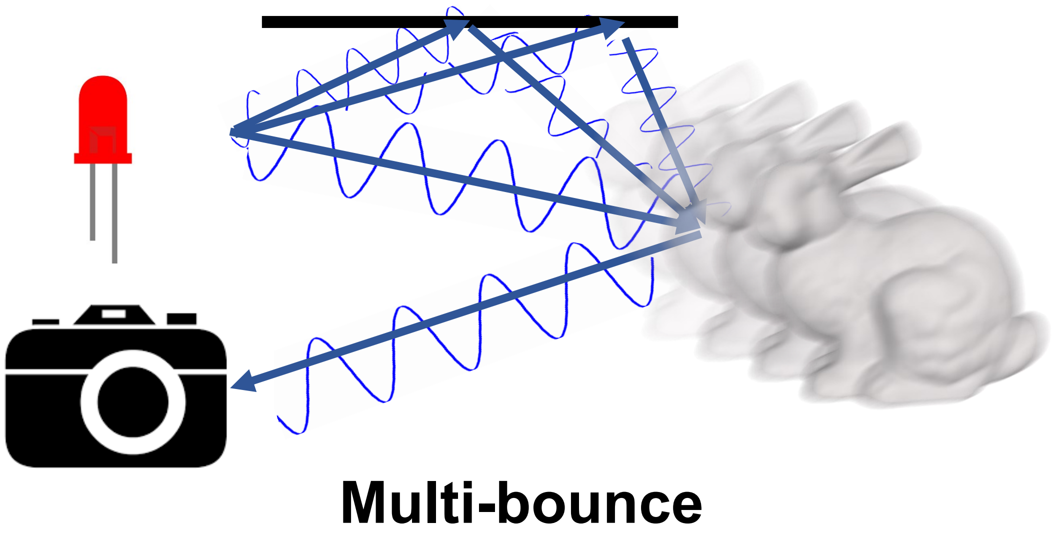

Uniform
Stratified
Ours
D-ToF Measurements
Standard Rendering
Reconstructed Radial Velocity
GT Radial Velocity

Why Physically-based Rendering Required?



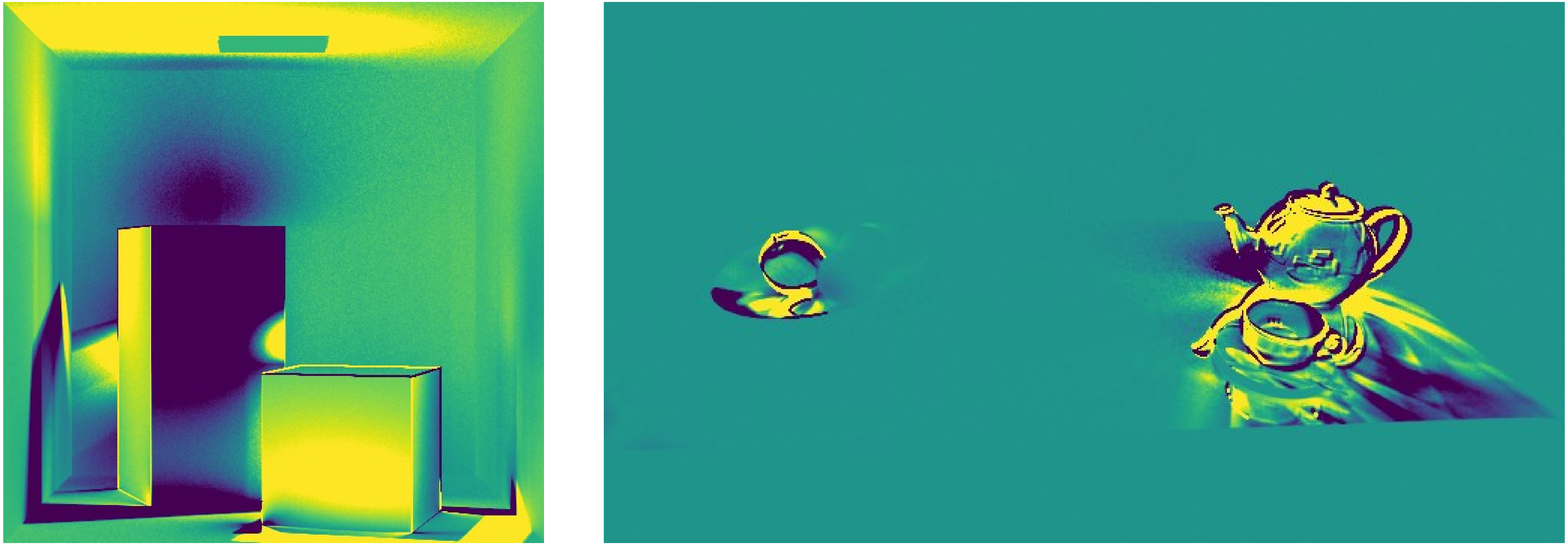


× Physically Not Accurate



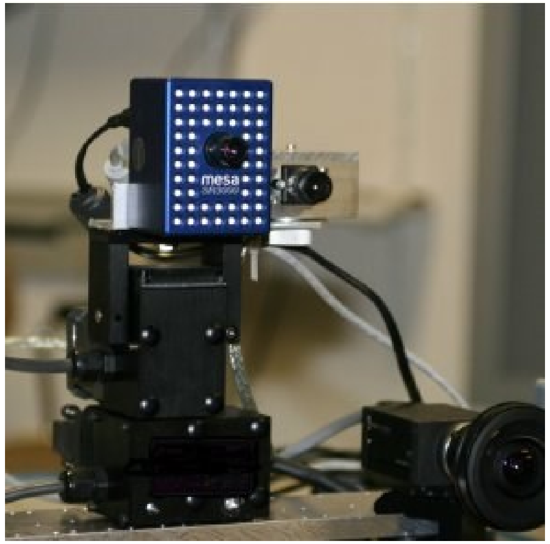
✓ Physically Accurate

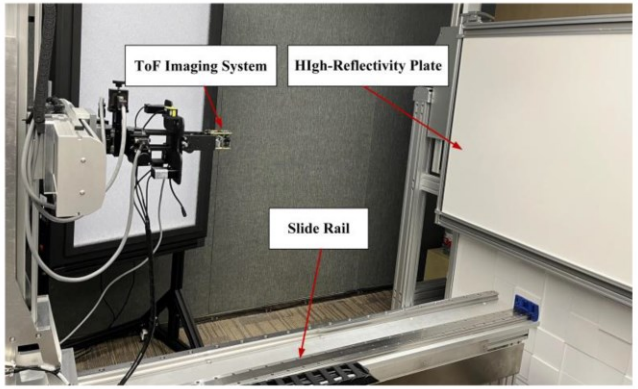
Why Physically-based Rendering Required?



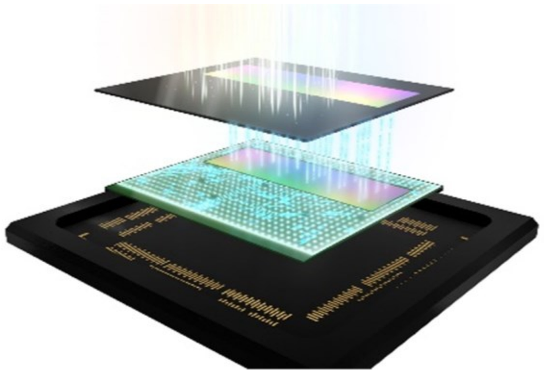
Extreme Multi-bounce Cases

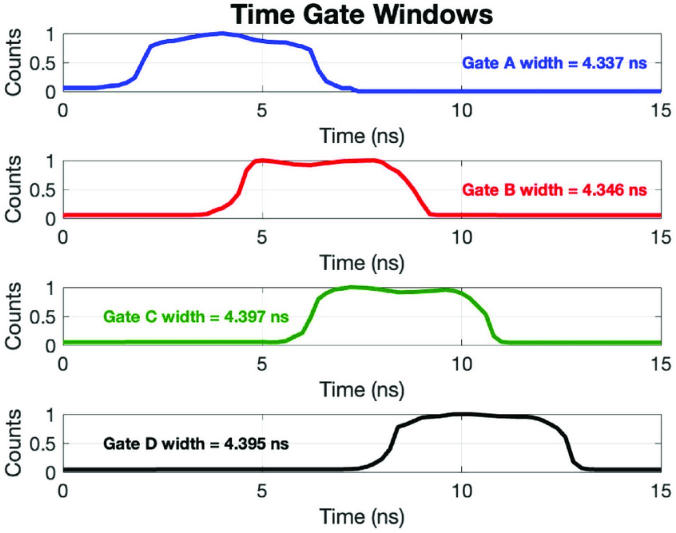
Why Physically-based Rendering Required?



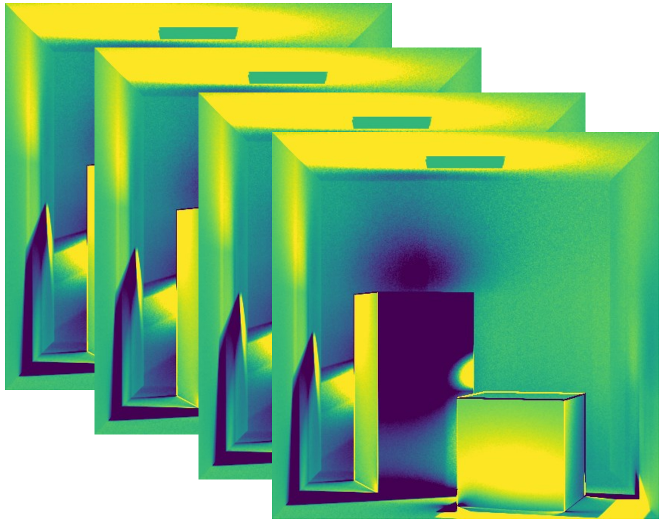


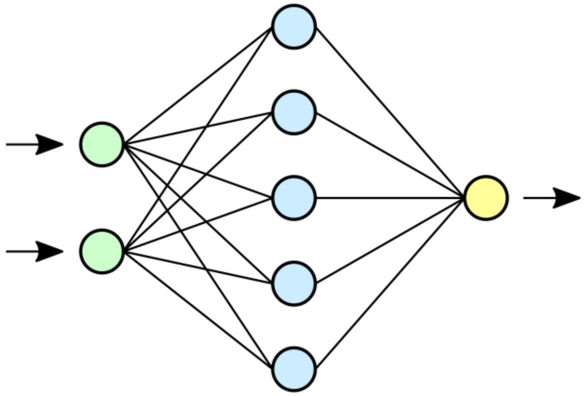
Imaging System Design & Simulation





Sensor Design





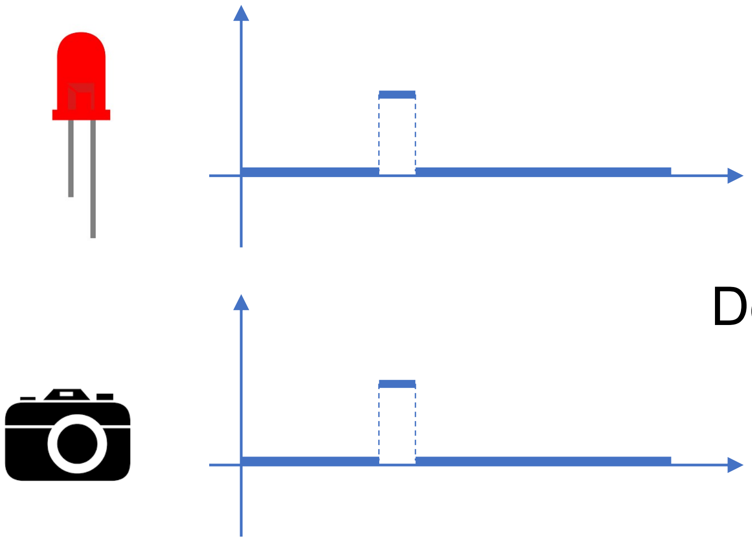
Large Dataset Generation for Machine Learning

Two Challenges of Doppler Time-of-Flight Rendering

ToF (Transient / Time Gated) Rendering

[Jarabo 2012, 2014; Marco 2017, 2019; Pediredla 2019; Liu 2022]

Static Scene

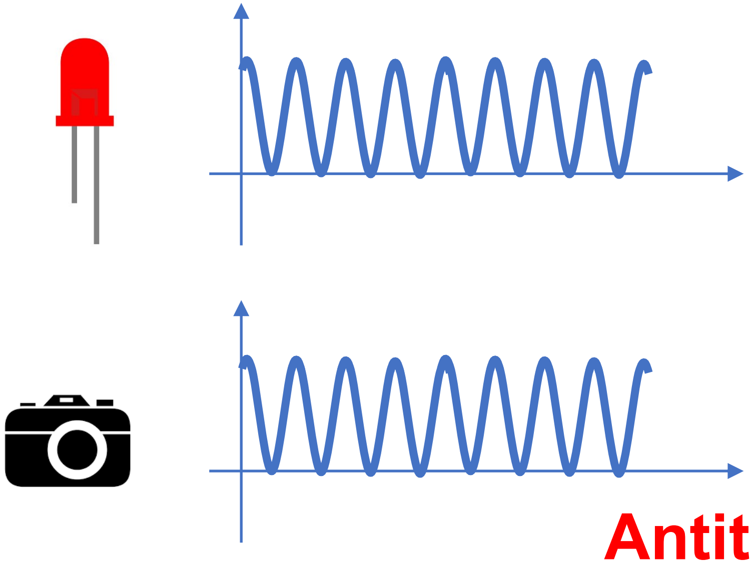


Delta-Modulation

Doppler ToF Rendering

Dynamic Scene

Path Correlation

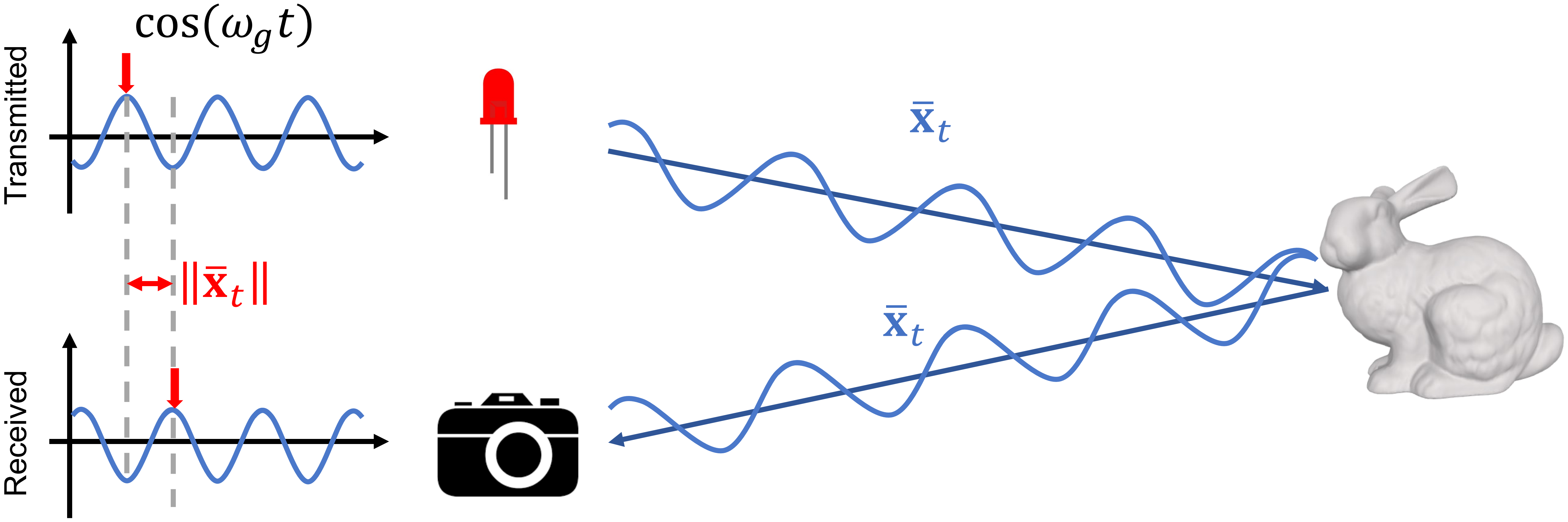


High-frequency Modulation (10-1000 MHz)

Antithetic Sampling

Simulation of D-ToF Camera

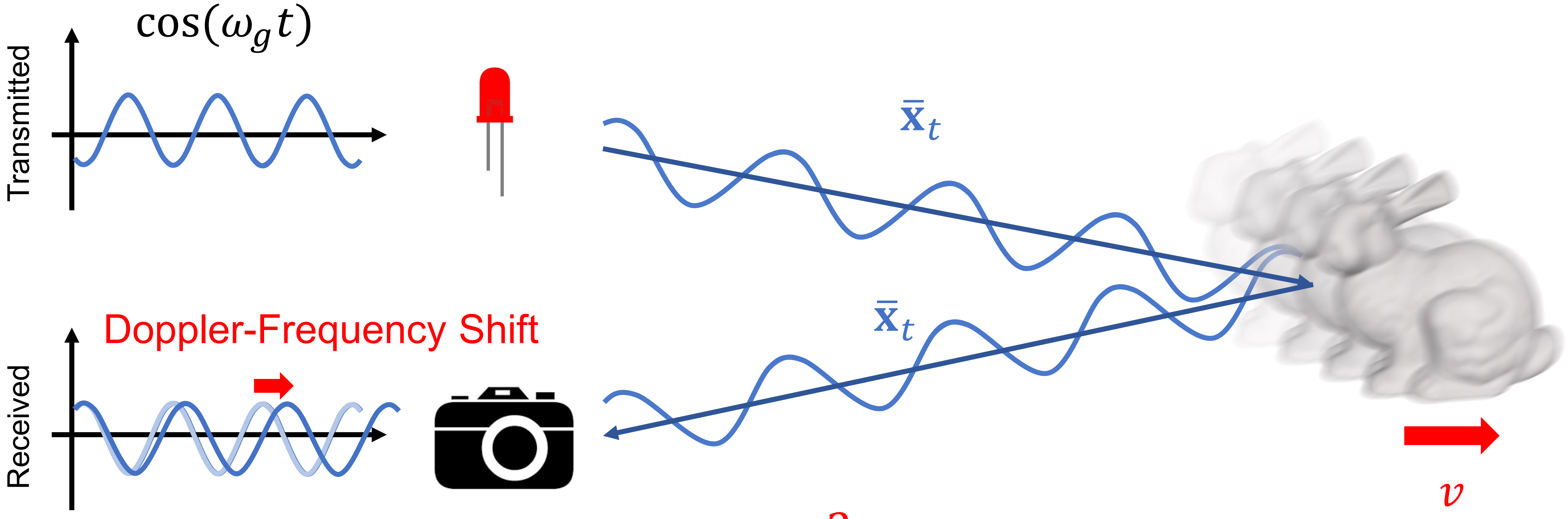
Backgrounds: D-ToF Camera



$$\cos\left(\omega_g(t - \|\bar{\mathbf{x}}_t\|)\right)$$

Time-of-Flight

Backgrounds: D-ToF Camera

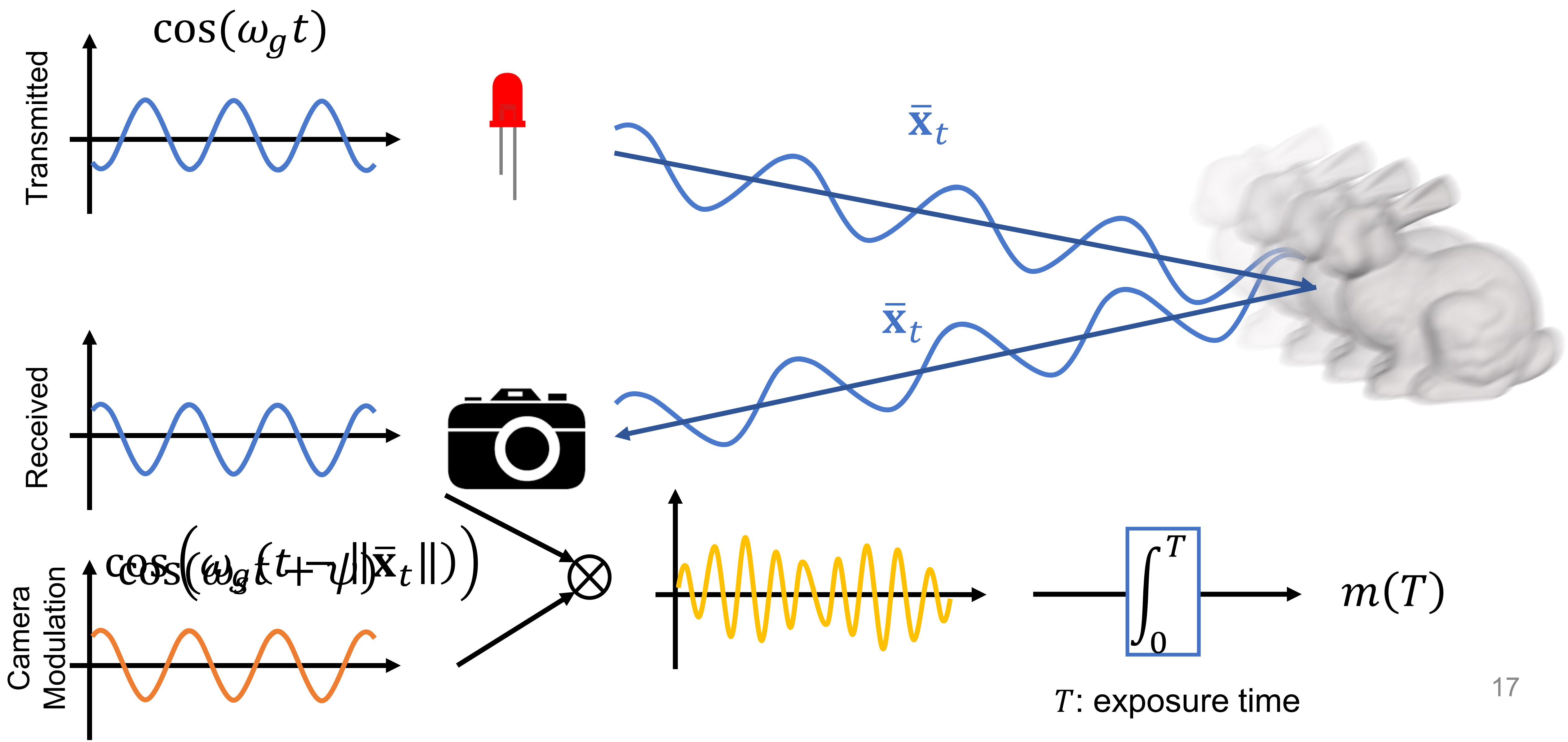


$$\cos\left(\omega_g(t - \|\bar{\mathbf{x}}_t\|)\right)$$

$$\|\bar{\mathbf{x}}_t\| = \|\bar{\mathbf{x}}_0\| + \frac{2v}{c}t$$

$$\omega_g \to \omega_g - \frac{2v}{c}$$

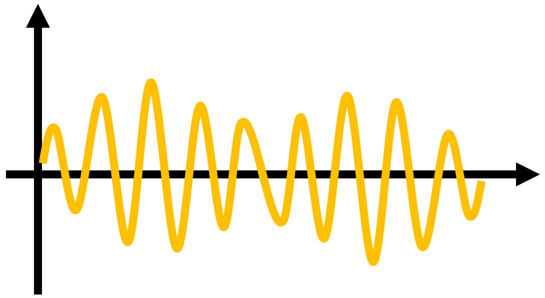
Backgrounds: D-ToF Camera



Backgrounds: D-ToF Measurement

$$A = \omega_s t + \psi$$

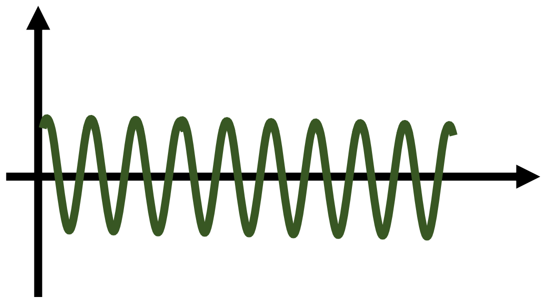
$$B = \omega_g (t - \|\bar{\mathbf{x}}_t\|)$$



$\cos A \cos B$

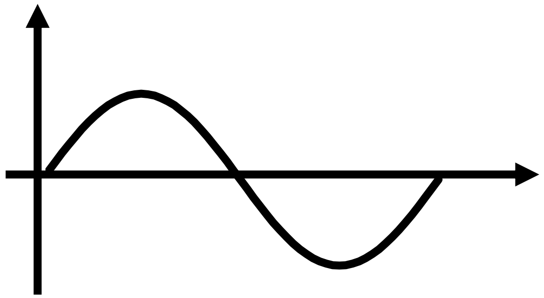
=

$\cos(A + B)$



$\int_0^T$ → **Vanish**

+



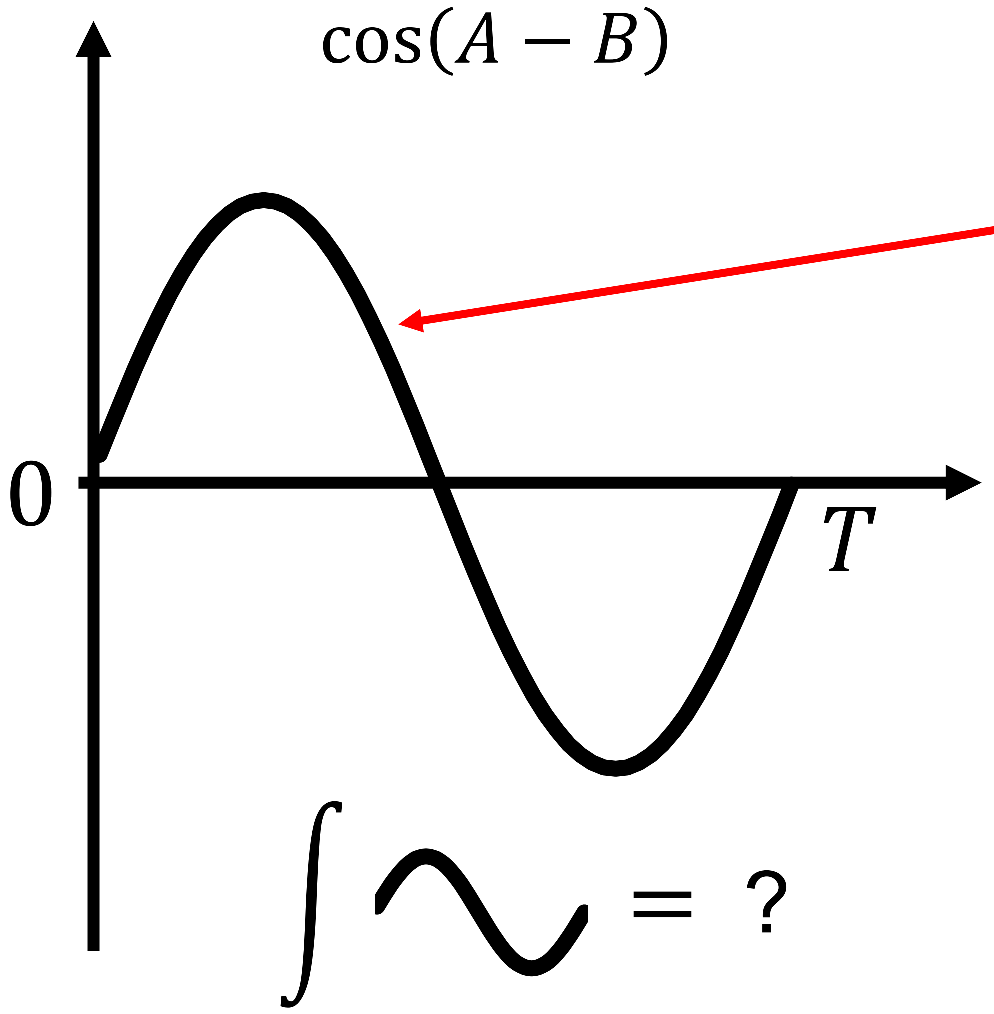
$\int_0^T$ → **?**

$\cos(A - B)$

Backgrounds: D-ToF Measurement

$$A = \omega_s t + \psi$$

$$B = \omega_g (t - \|\bar{\mathbf{x}}_t\|)$$



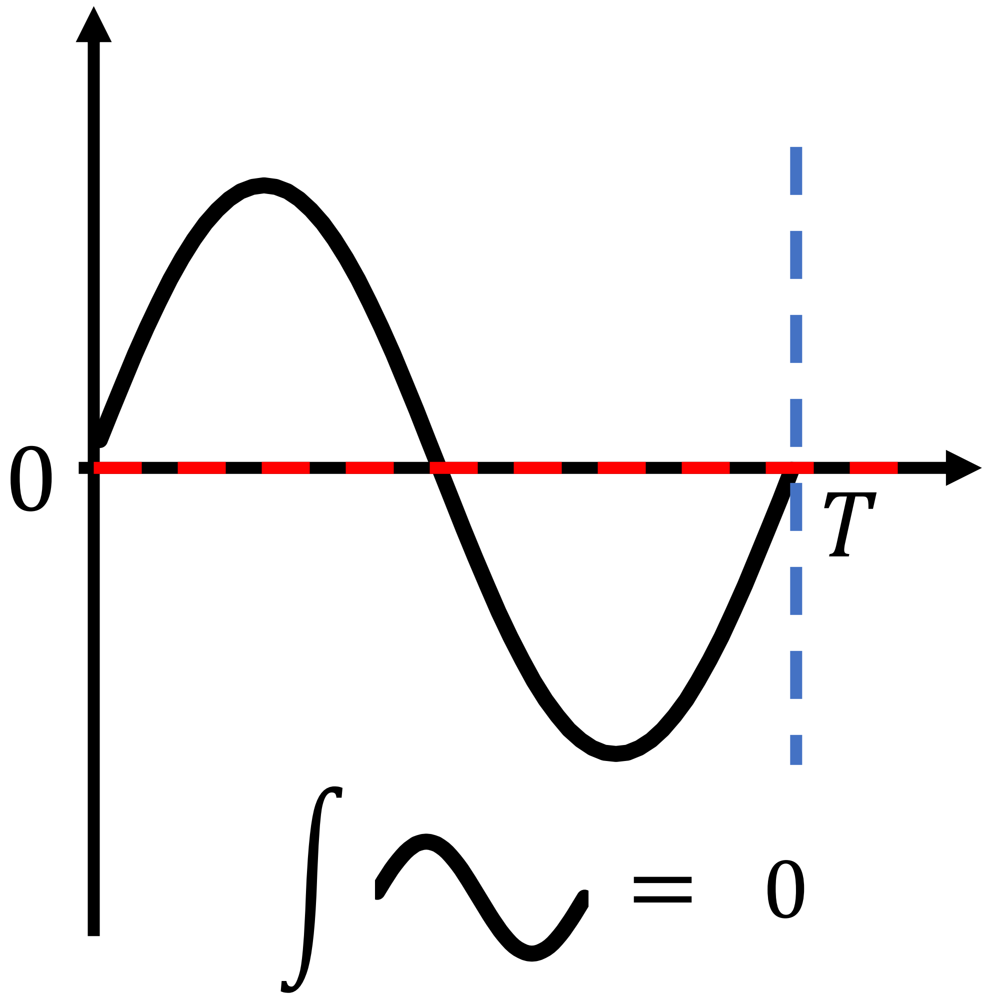
$$\boxed{\cos\left((\omega_s - \omega_g)t + \psi - \omega_g \|\bar{\mathrm{x}}_t\|\right)}$$

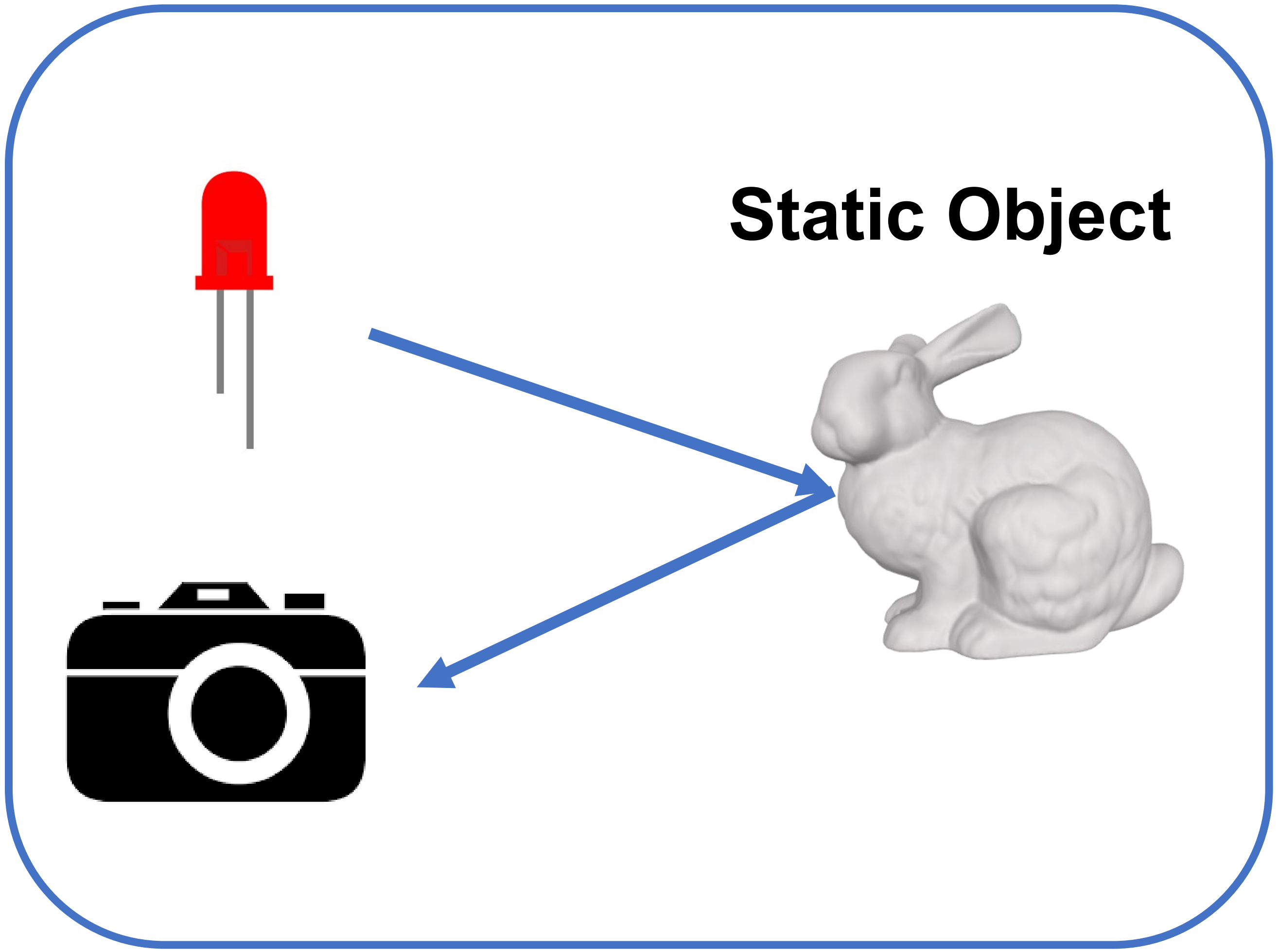
Heterodyne frequency $\omega_{\mathrm{d}} = \omega_s - \omega_g$

One-period sinusoidal [Heide 2015]

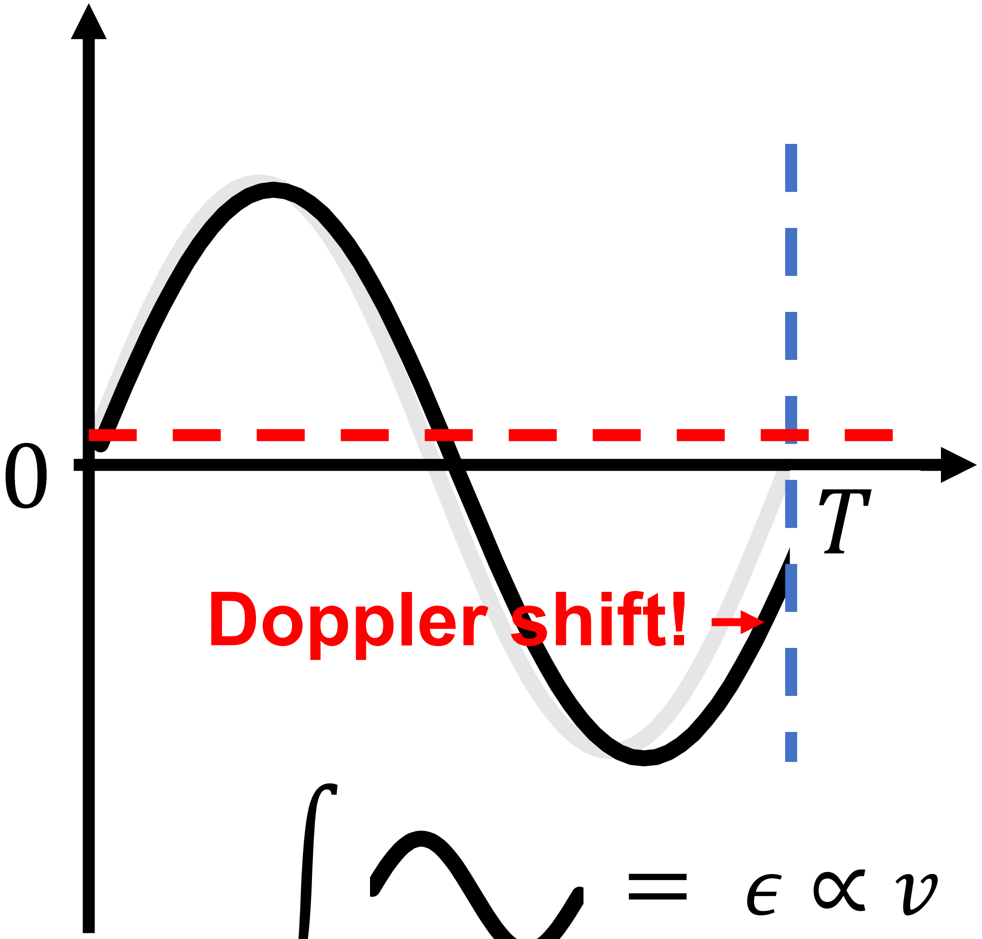
$$\omega_{\mathrm{d}} = \frac{2\pi}{T}$$

D-ToF Measurement – Static Object



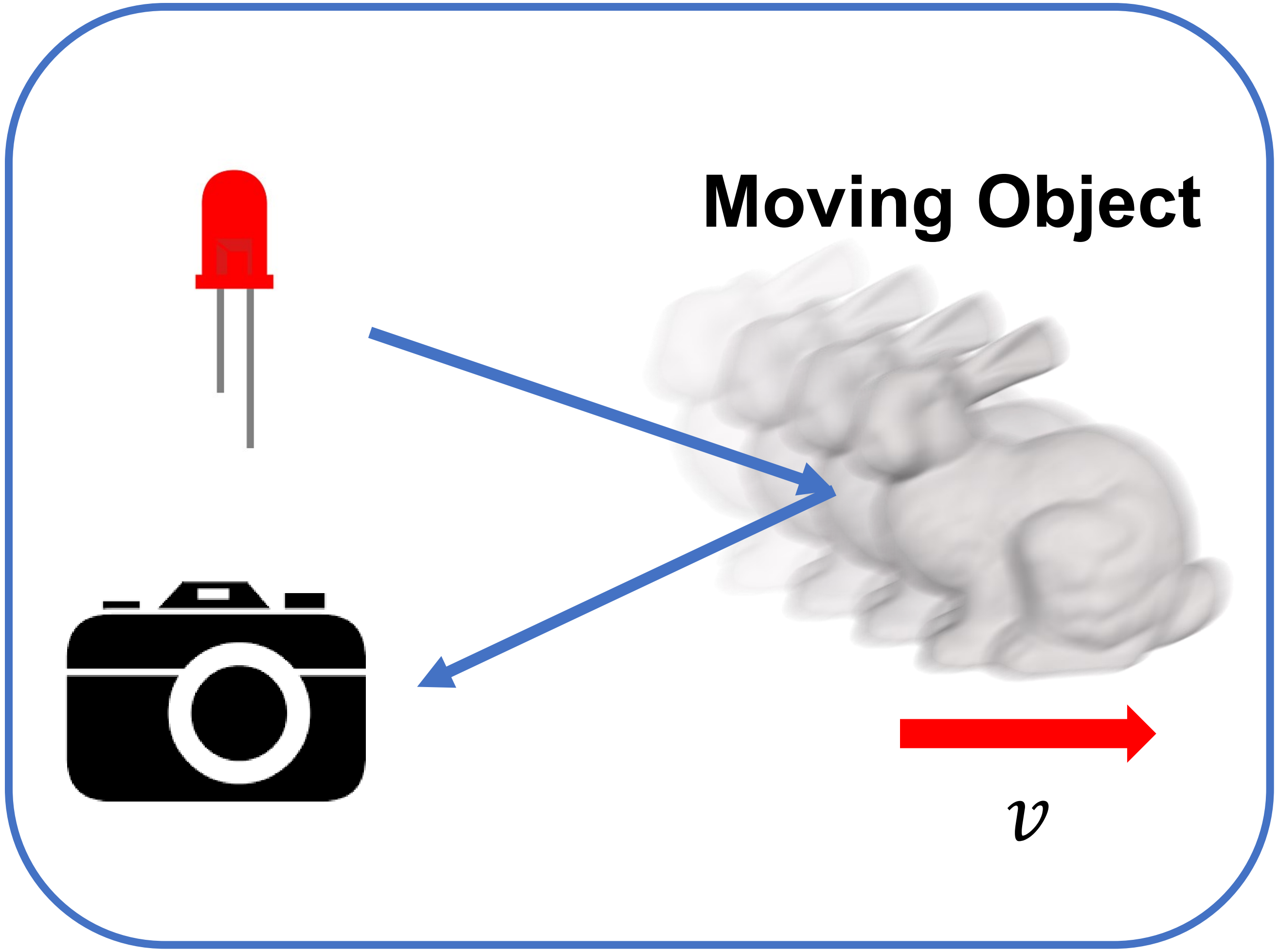


D-ToF Measurement – Dynamic Object



$$\int \sim \; = \; \epsilon \propto v$$

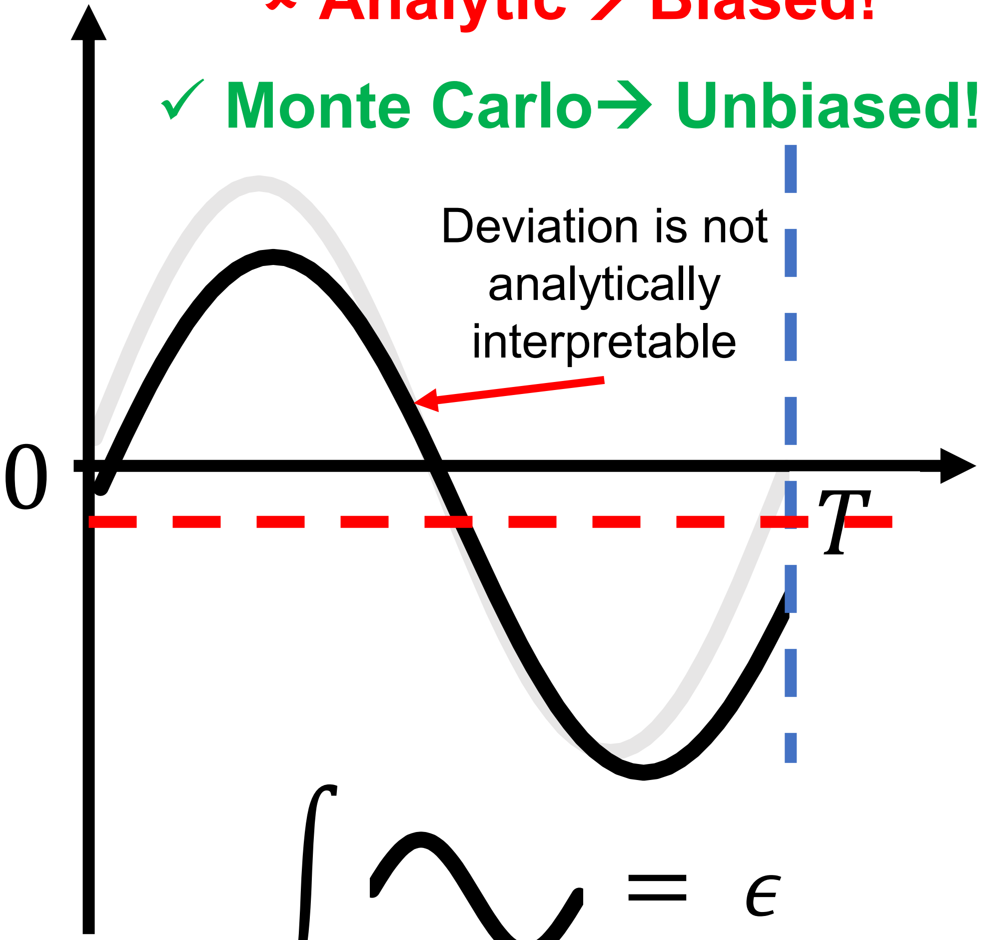
small value



D-ToF Measurement – General Cases

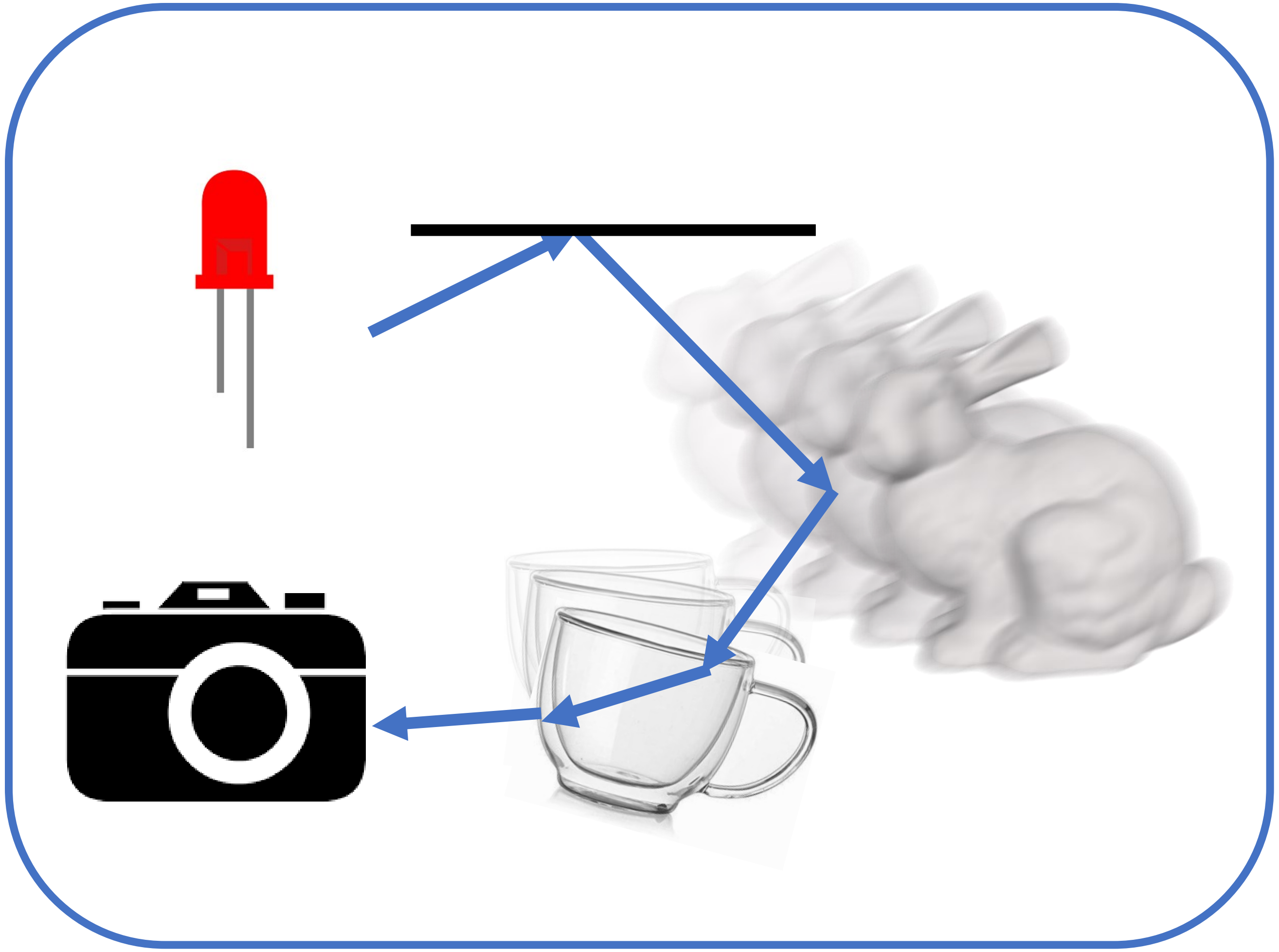
× Analytic→ Biased!

✓ Monte Carlo→ Unbiased!

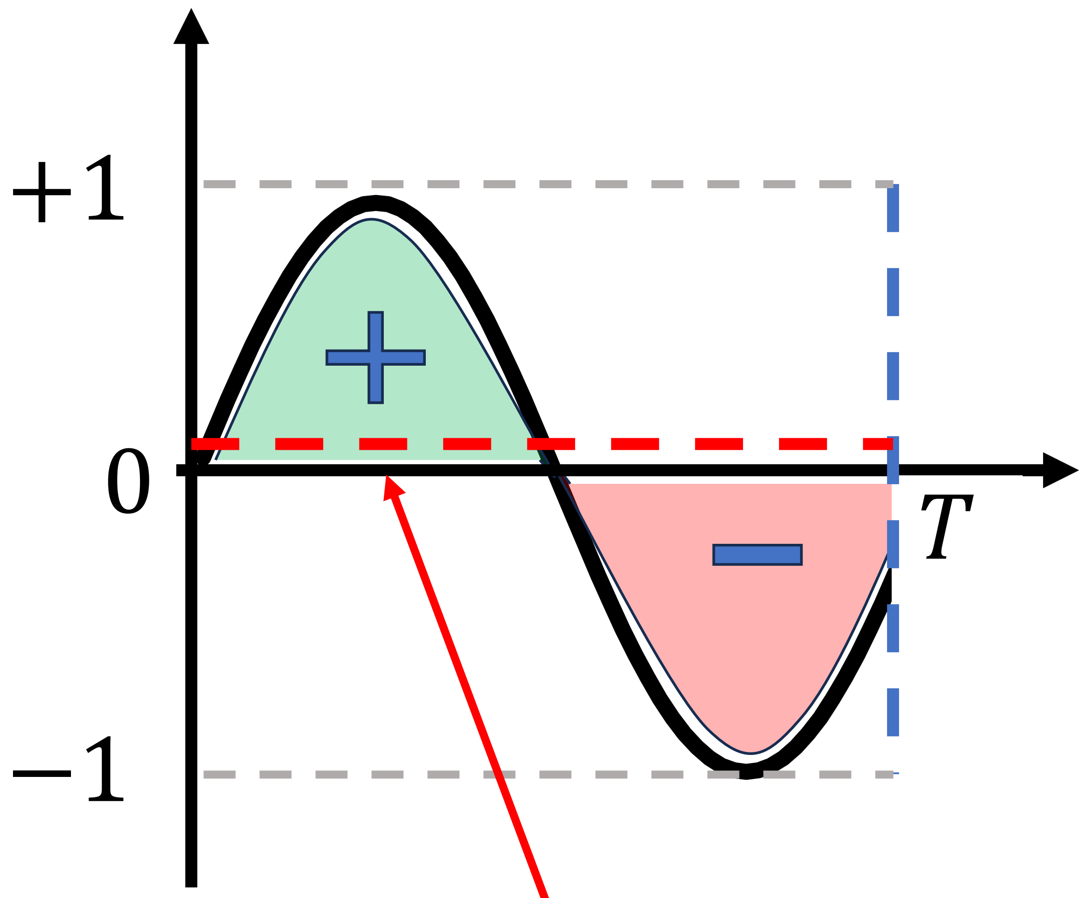


$$\int \sim \ = \ \epsilon$$

small value

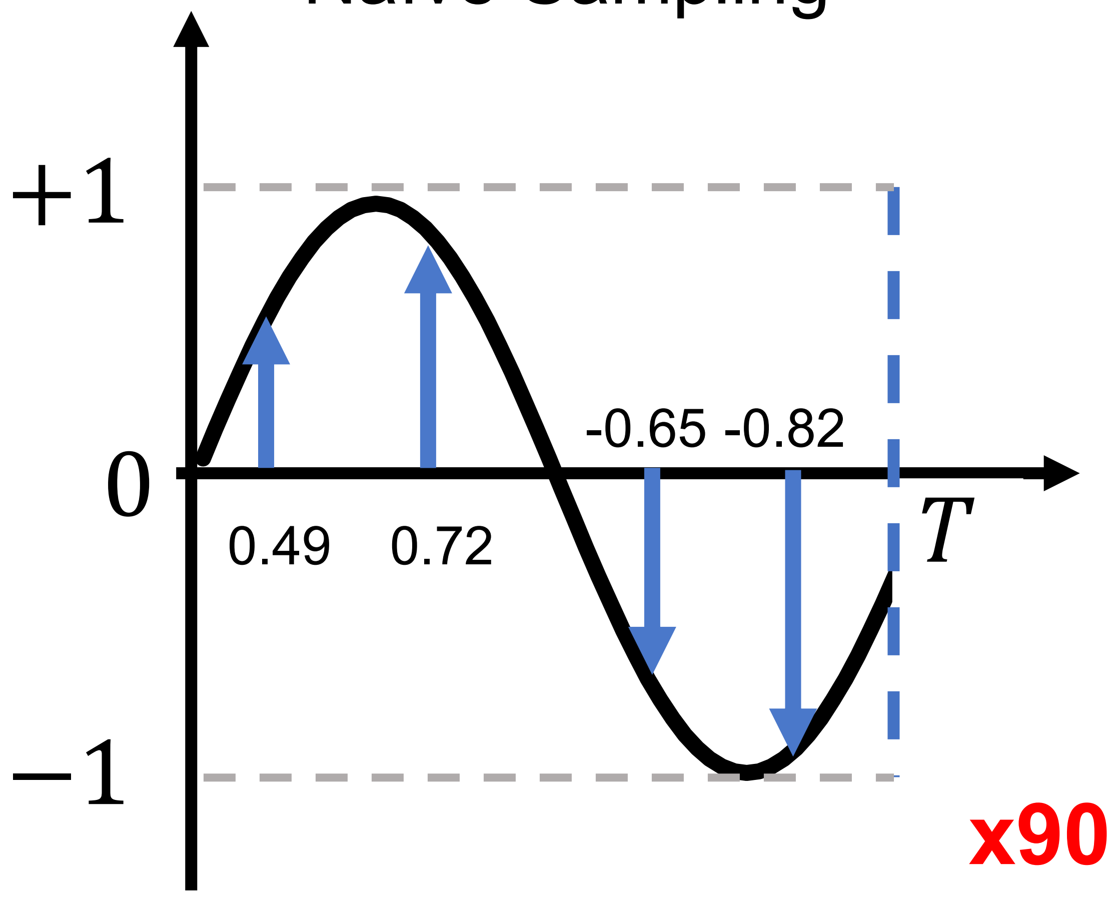


Monte Carlo Evaluation for D-ToF Measurement



GT of $m(T)$: 0.001

Naïve Sampling



x90

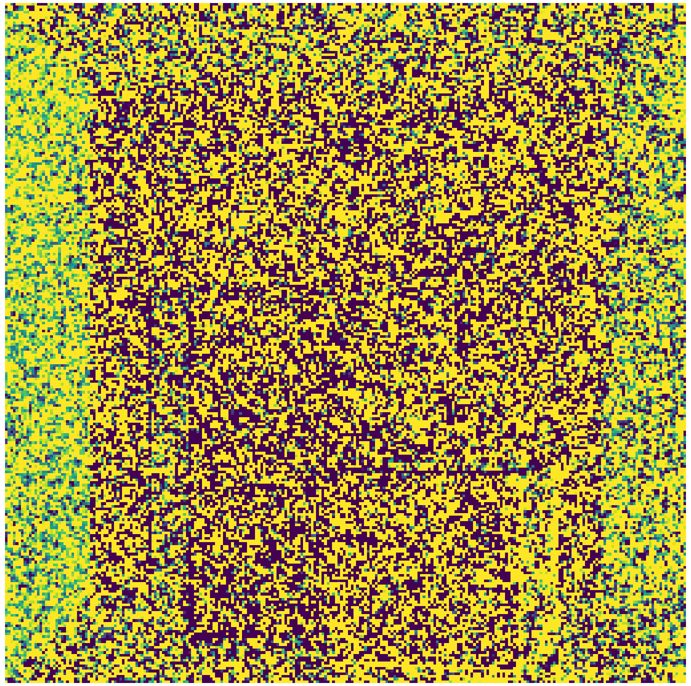
$\langle m(T) \rangle$: (0.49+0.72-0.65-0.82) / 4 = -0.09

Extremely Low SNR

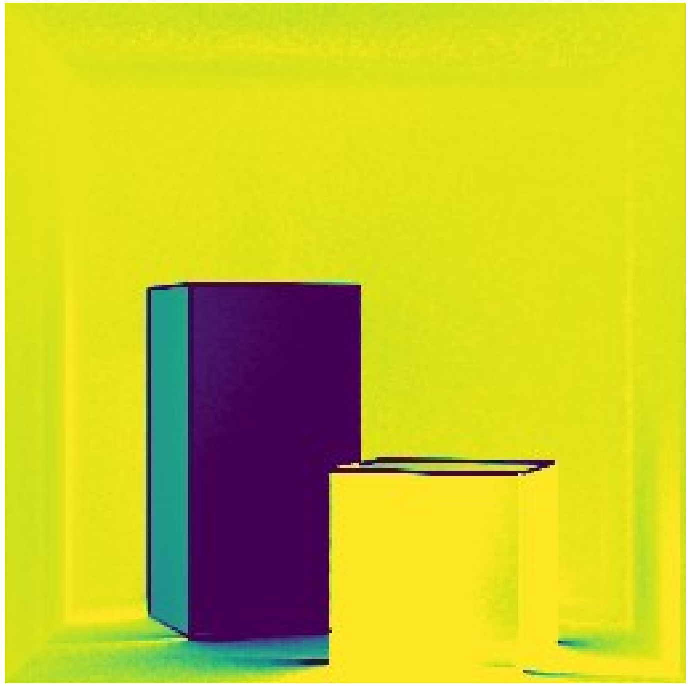
Evaluation of the Integrand using Monte Carlo Method

Typical Motion Blur Scene

D-ToF Rendering

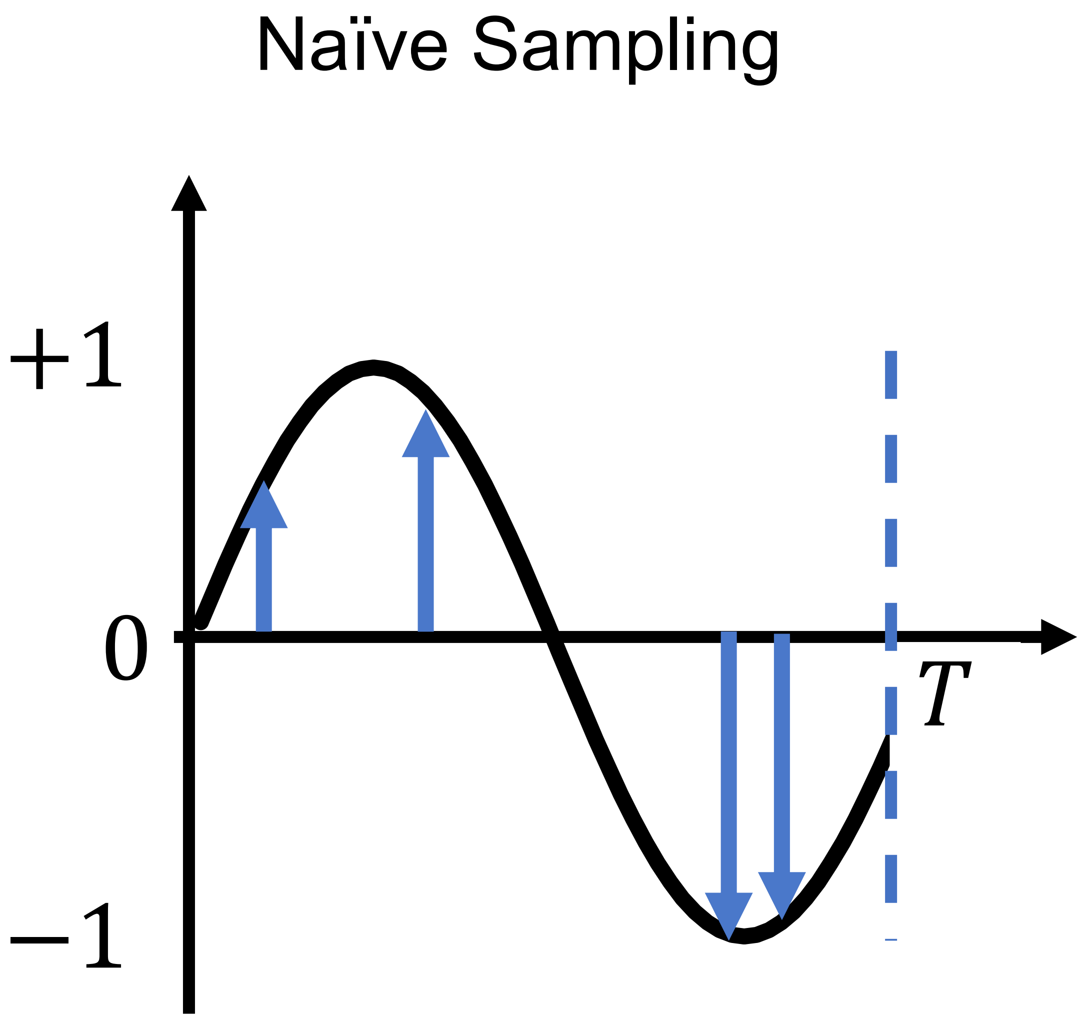


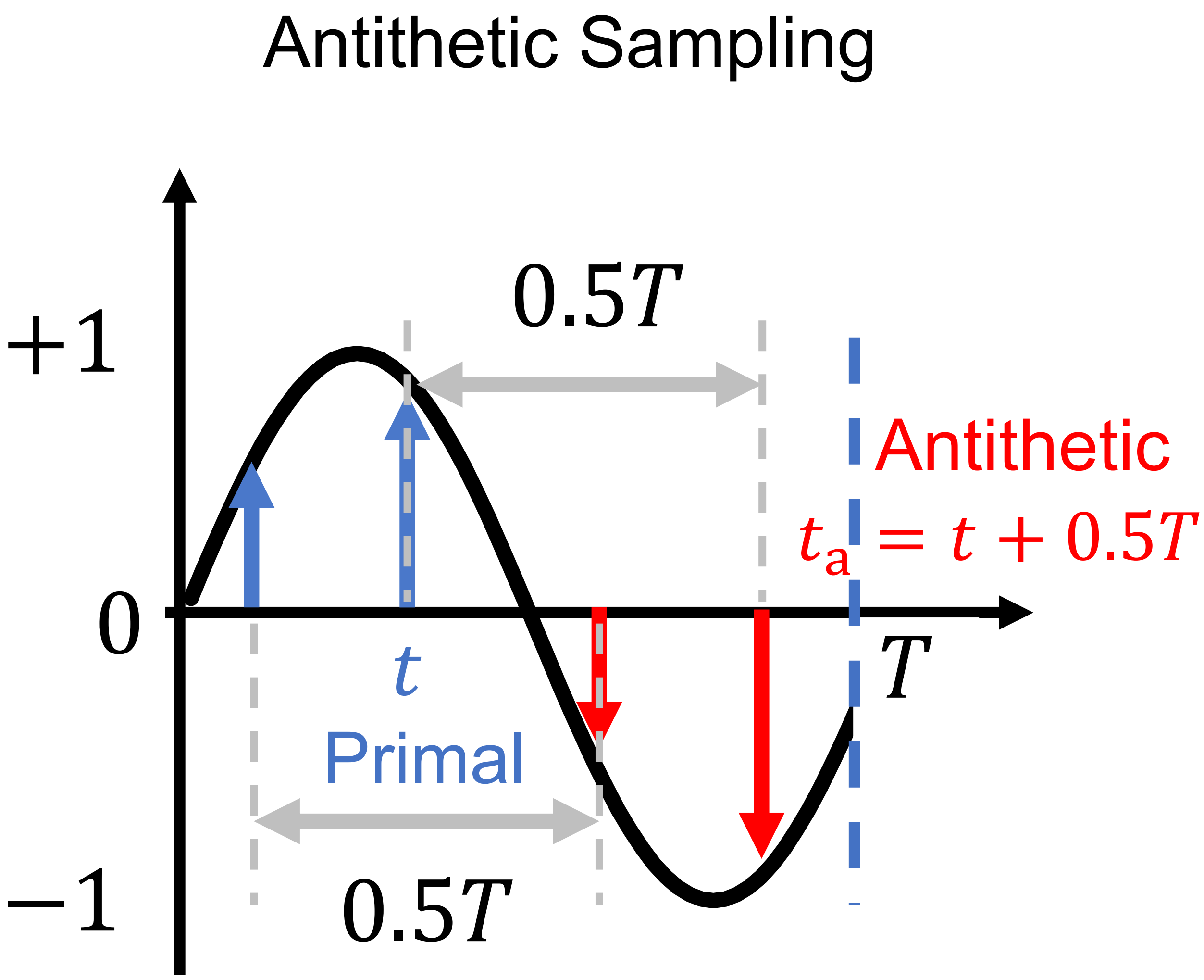
Motion Blur with
Uniform Time Sampling



Ground Truth

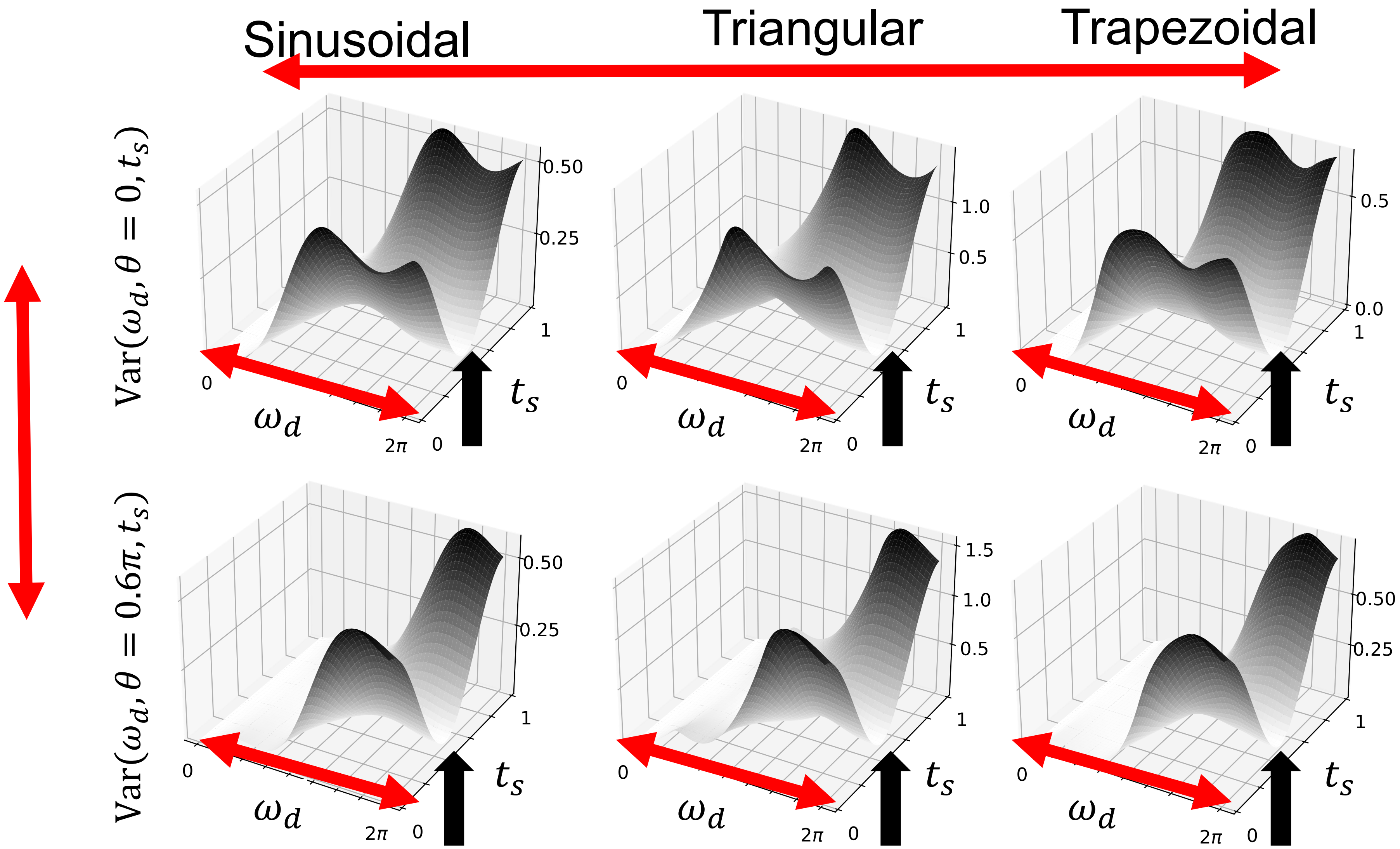
Proposed Method : Antithetic Time Sampling





$$\cos x + \cos(x + \pi) = 0$$

Proposed Method : Antithetic Time Sampling



$0.5T$ shift is useful regardless of

✓ **Frequency**

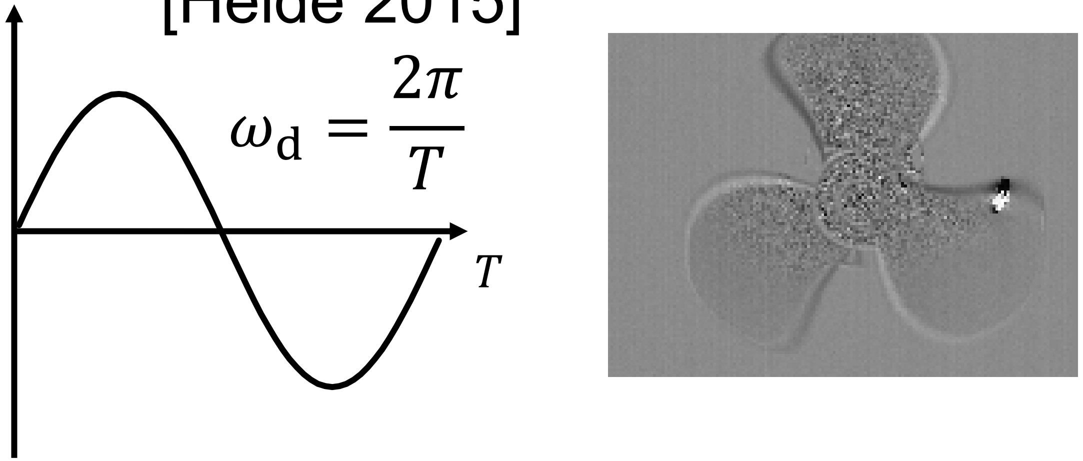
✓ **Phase Offset**

✓ **Waveform**
(only some)

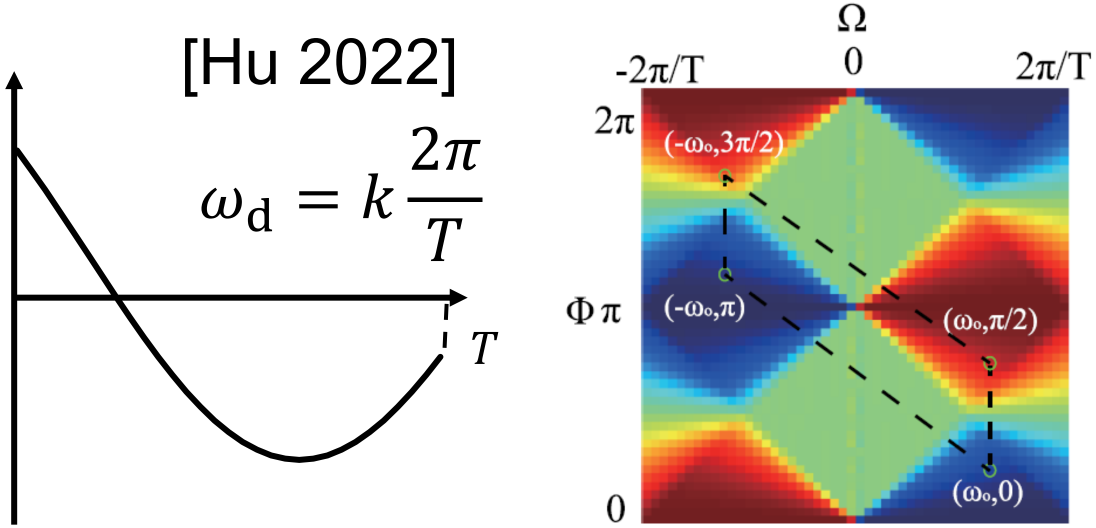
Mathematically provable!

Arbitrary **frequency** $0 < \omega_d < \frac{2\pi}{T}$

[Heide 2015]

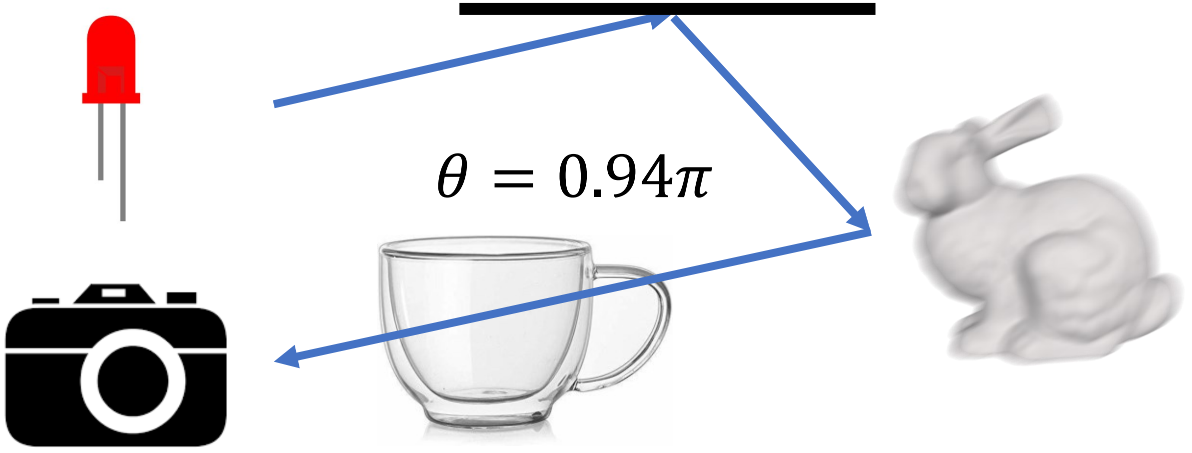


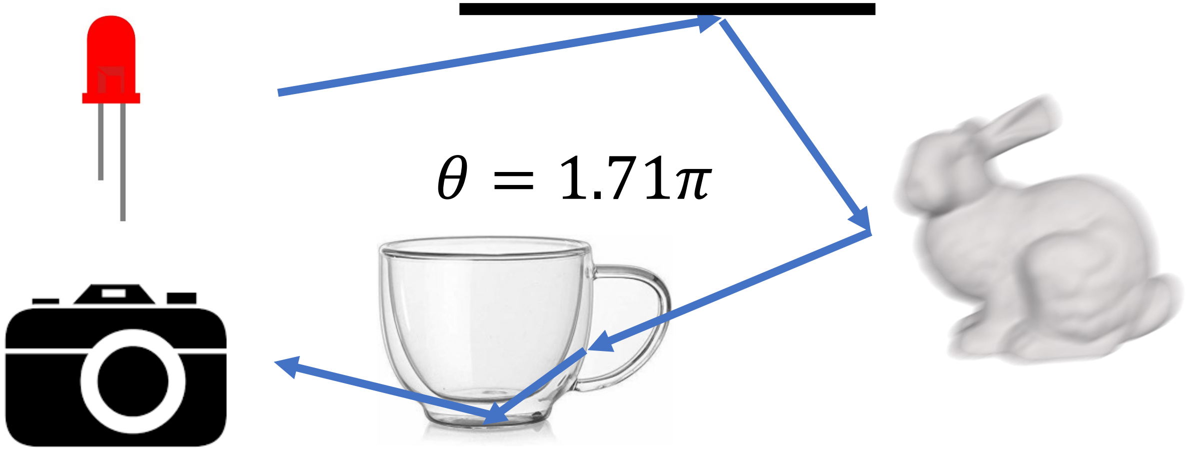
[Hu 2022]



✓ **Applicable to arbitrary modes!**

Arbitrary **phase offset** $0 < \theta < 2\pi$



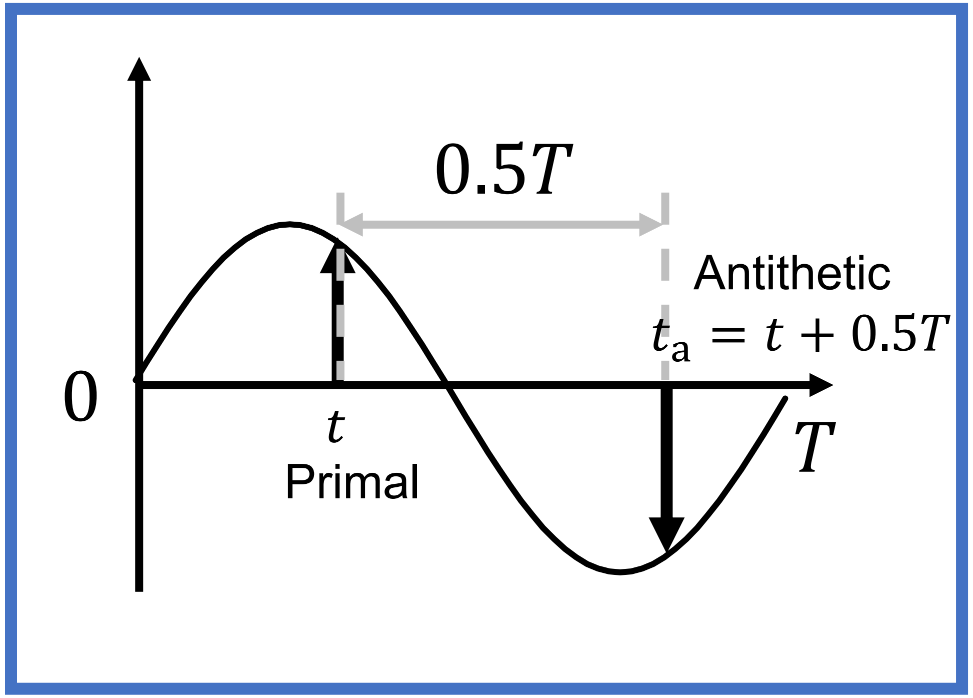


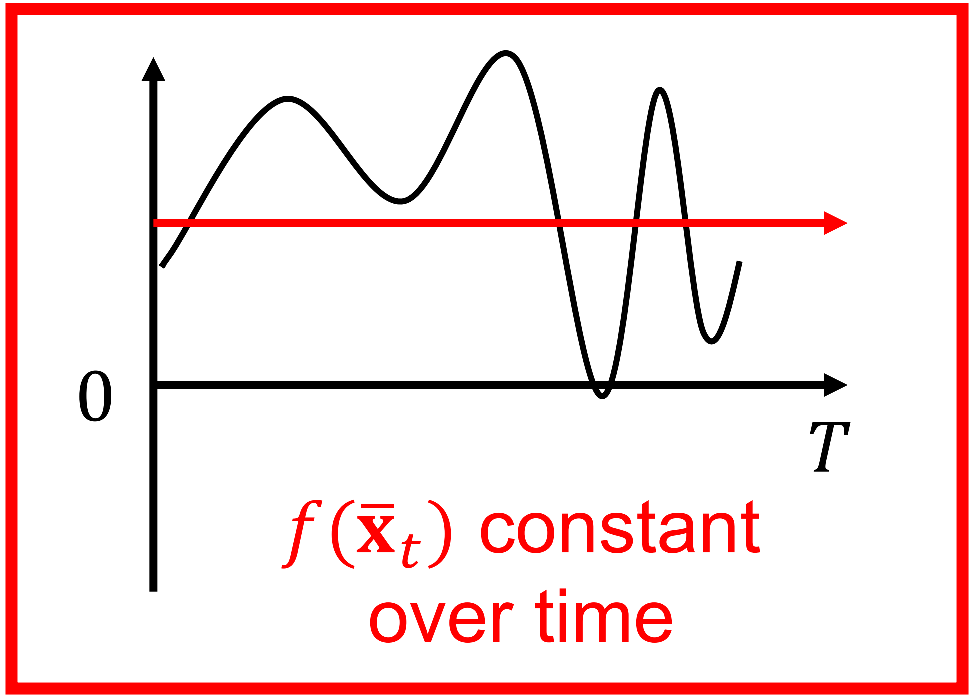
✓ **Useful for multi-bounce simulation**

Aligning Path over Time

Modulation **Path throughput**

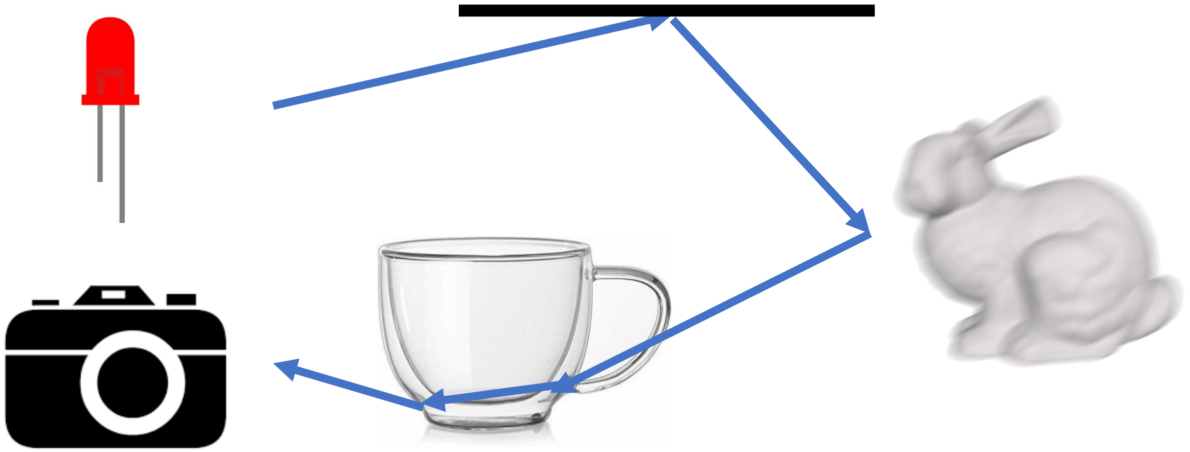
$$\int_0^T \int_{\mathcal{P}(t)} \cos(\omega_{\mathrm{d}} t + \psi - \omega_g \|\bar{\mathbf{x}}_t\|) \quad f(\bar{\mathbf{x}}_t)$$





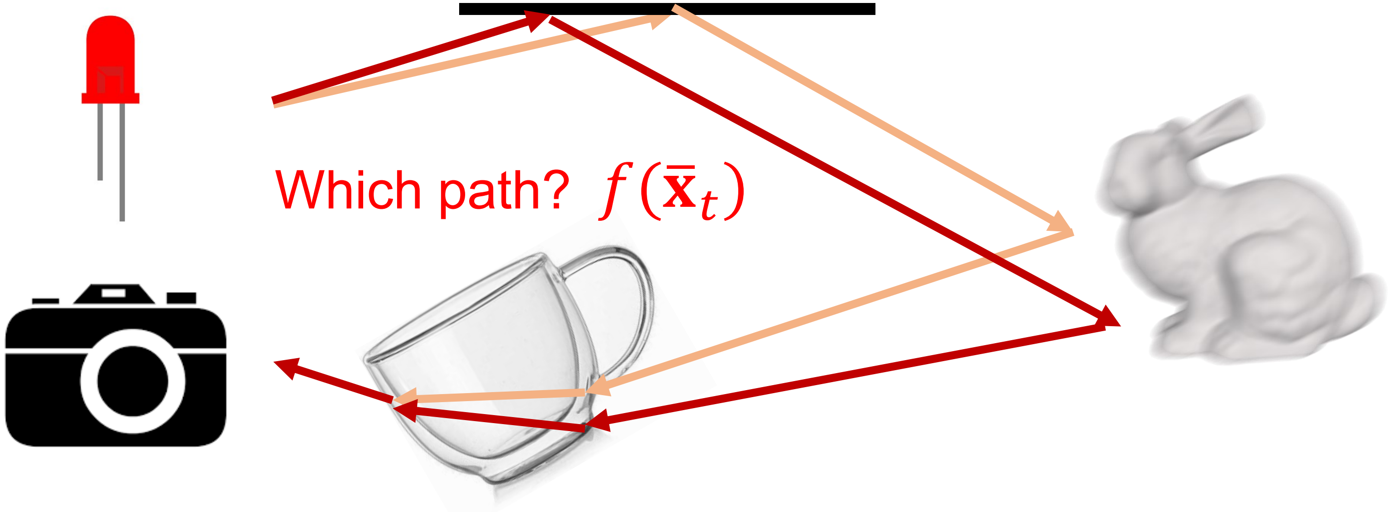
Aligning Path over Time

Time : t



Primal Path

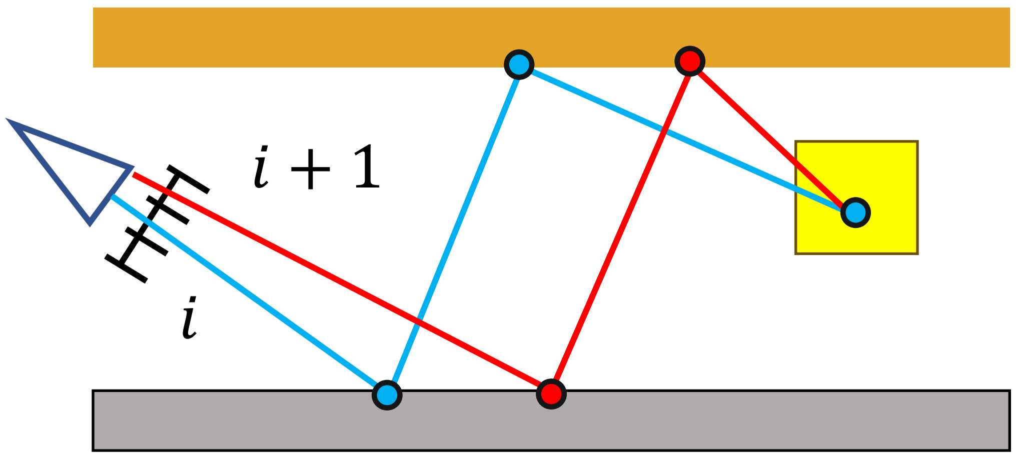
Time : $t_{\mathrm{a}} = t + 0.5T$



Antithetic Path

Aligning Path over Time using Shift Mapping

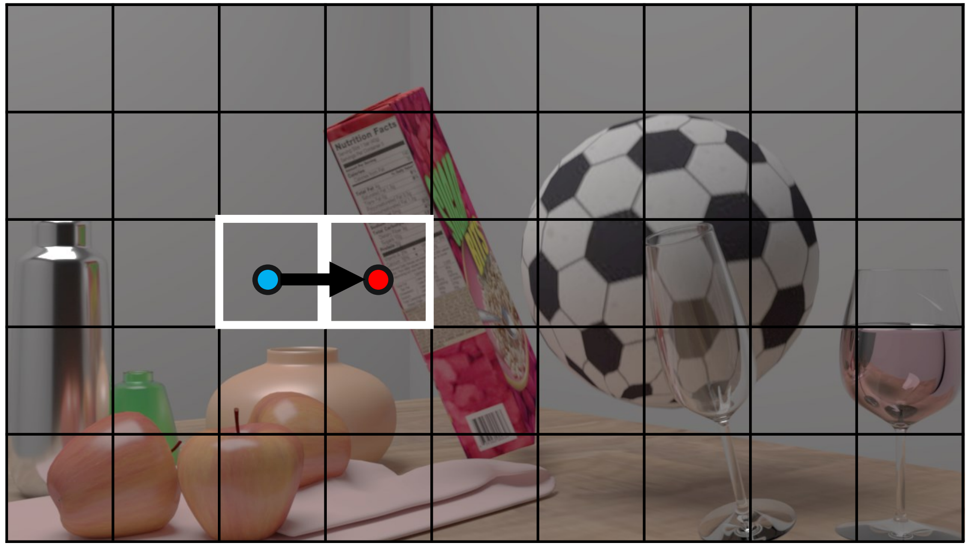
Shift Mapping
[Kettunen 2015]



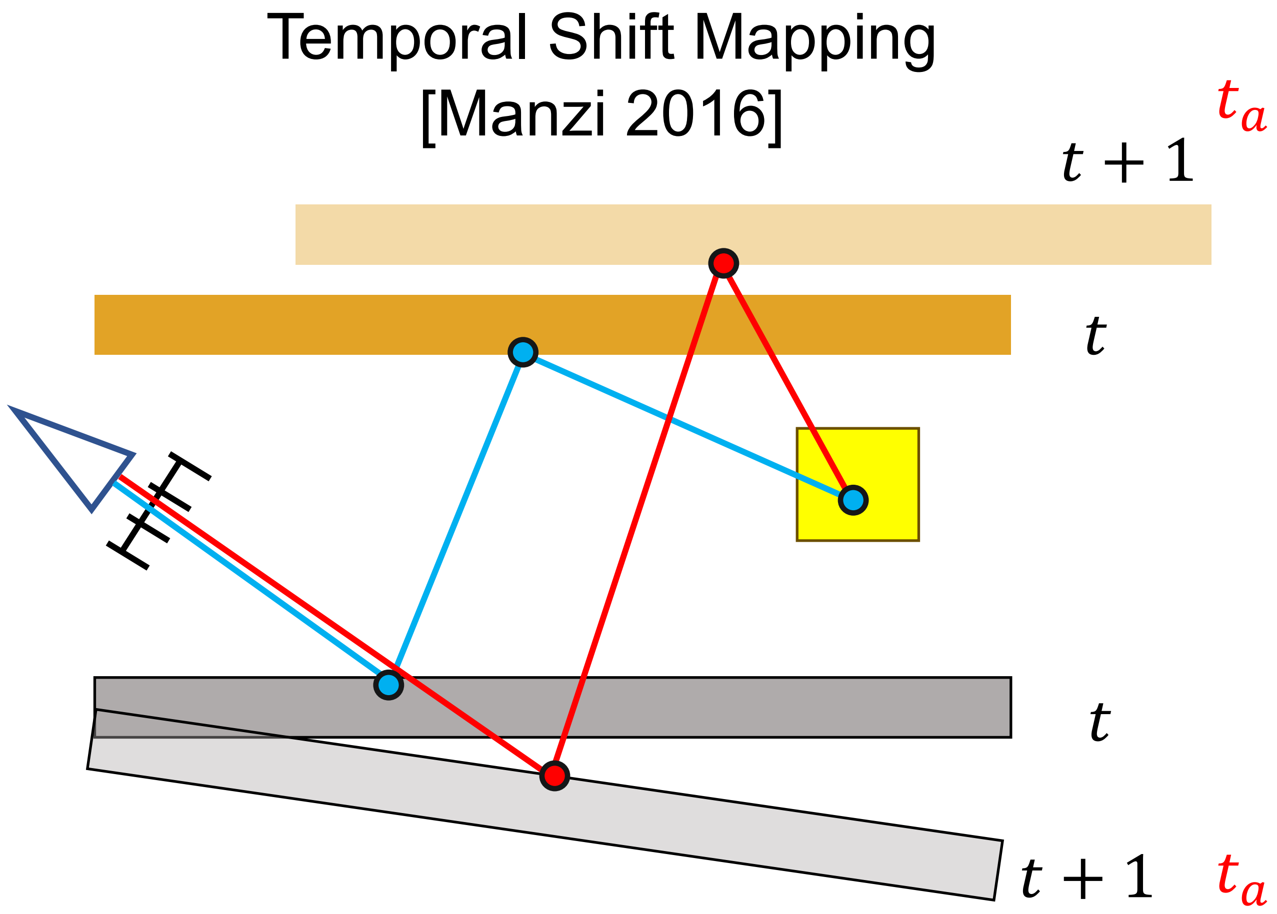
Base Path

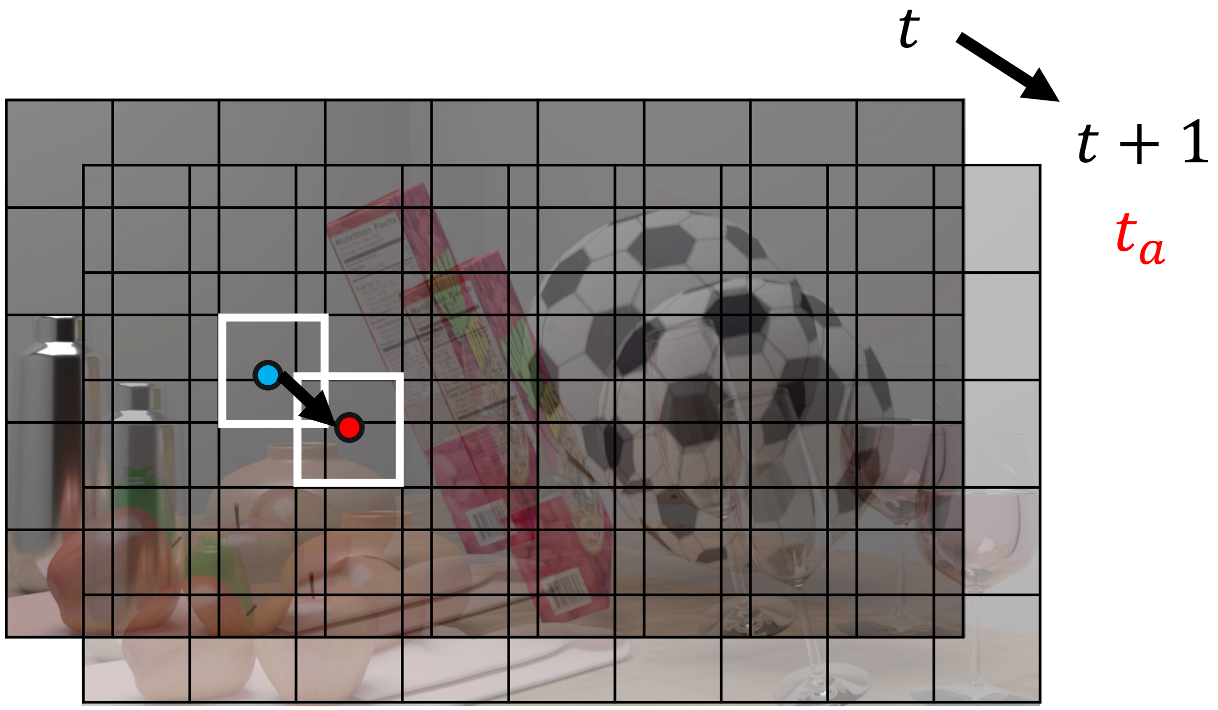
Offset Path

Gradient Domain Rendering



Aligning Path over Time using Shift Mapping

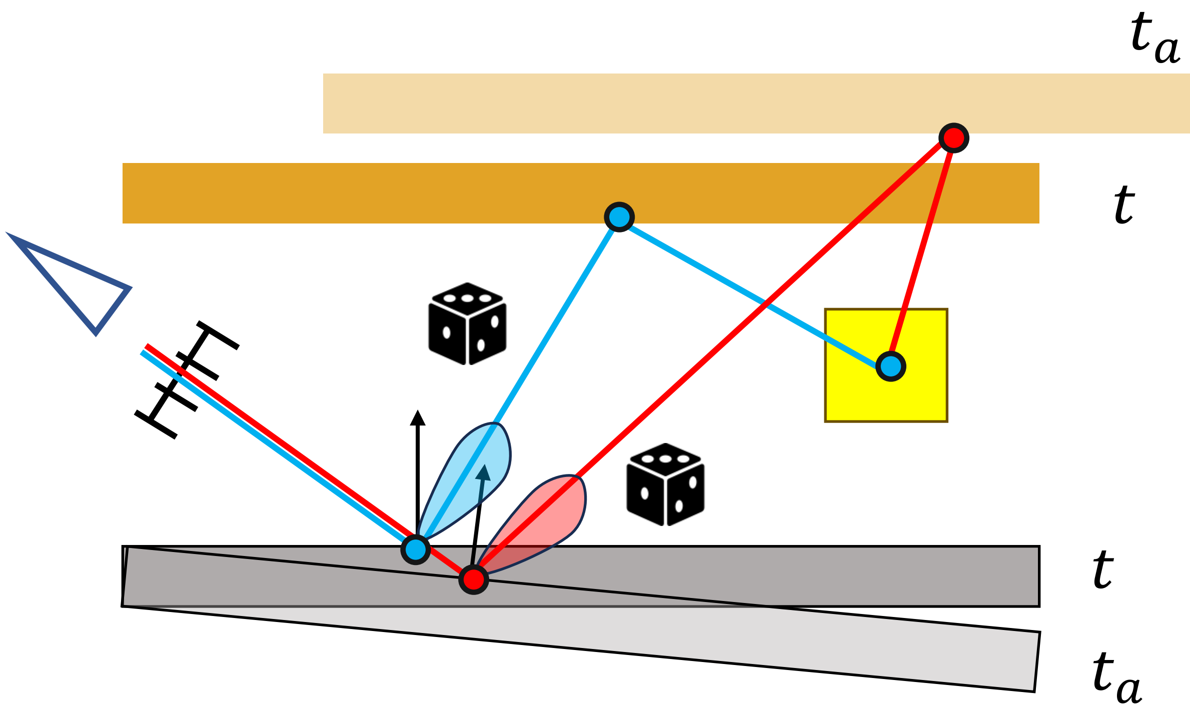




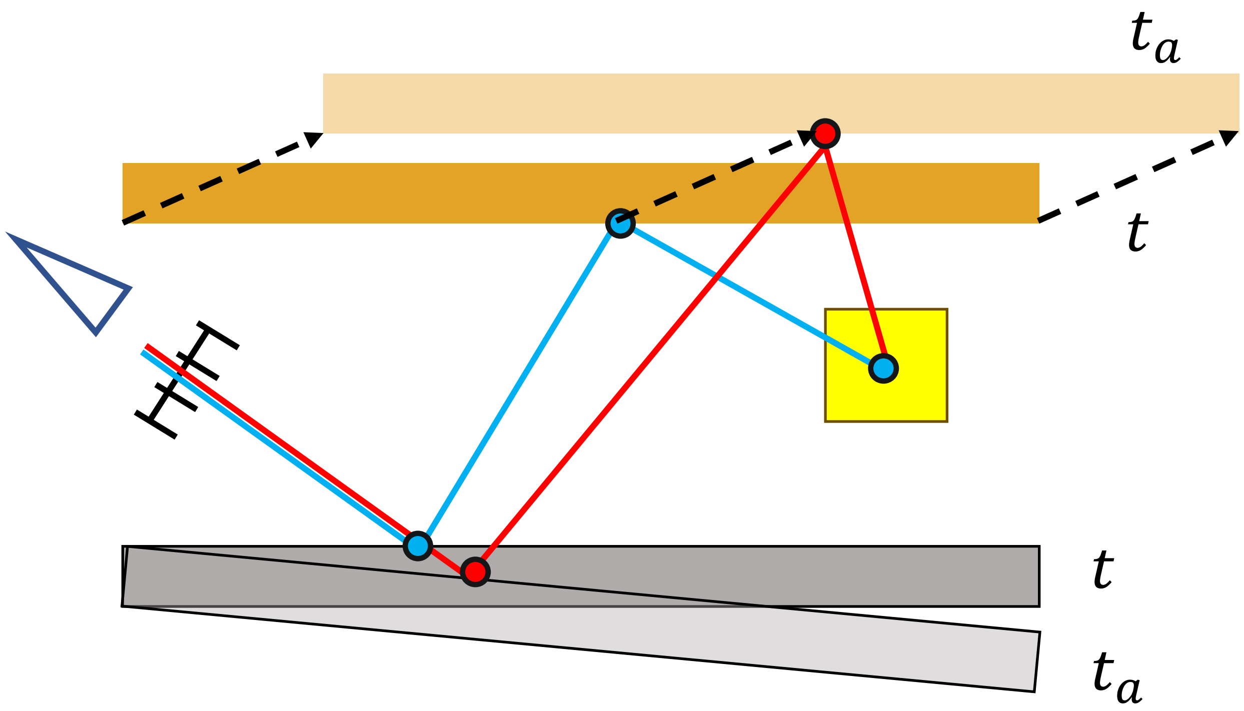
Base Path		*Primal Path*
Offset Path		*Antithetic Path*

Aligning Path over Time using Shift Mapping

Random Replay
[Hua 2019, Manzi 2016]



Path Reconnection
[Kettunen 2015]



Generally good for specular | **Generally good for diffuse**

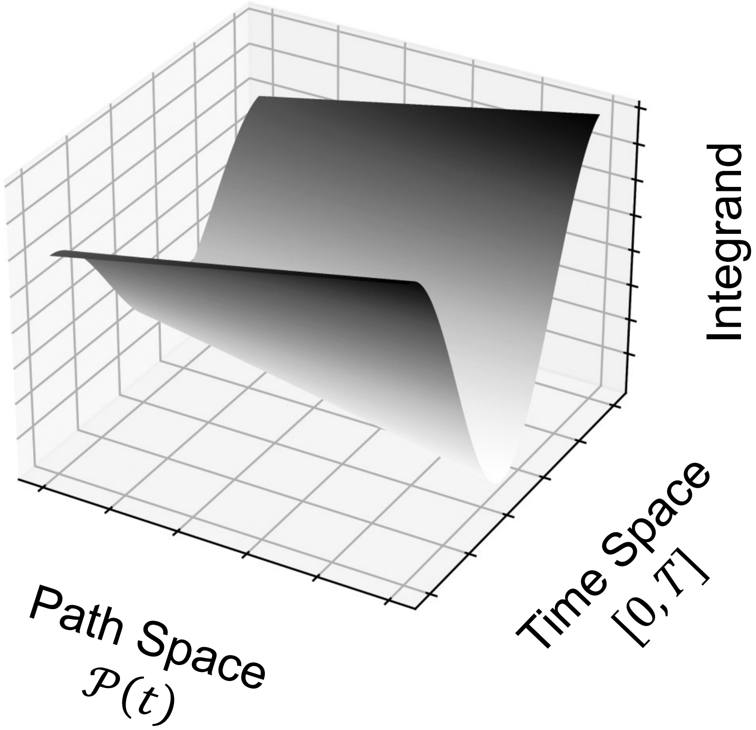
Mixed strategy based on surface material!

Proposed Sampling Strategy Overview

Goal : Integrate

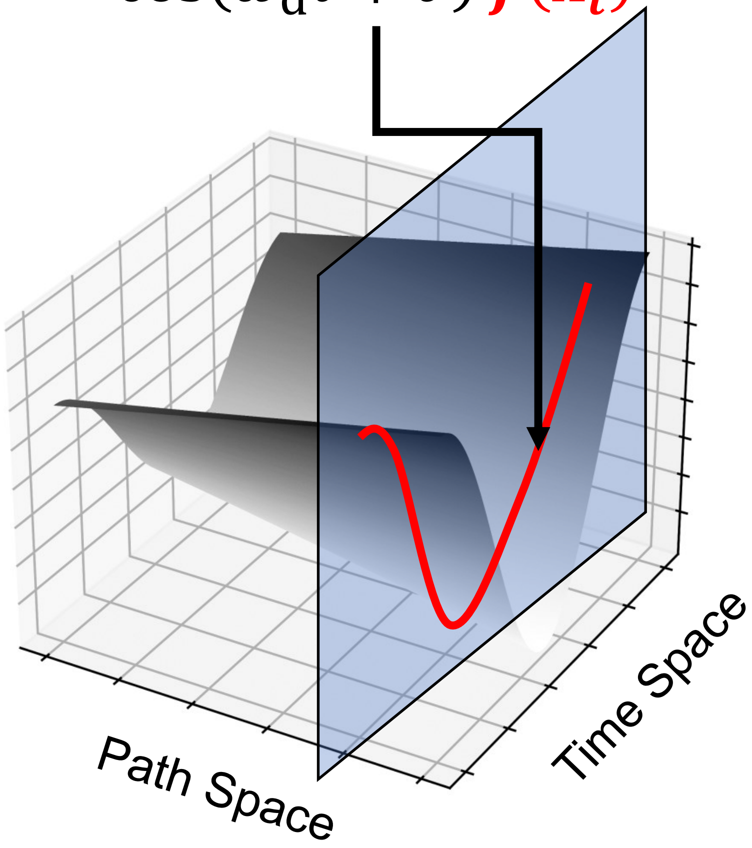
$$\cos(\omega_\mathrm{d} t + \theta)\, f(\bar{\mathbf{x}}_t)$$

$$\theta = \psi - \omega_g \|\bar{\mathbf{x}}_t\|$$



Constant!

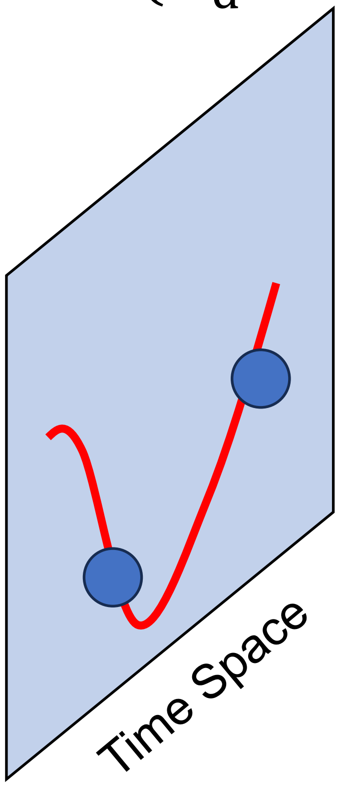
$$\cos(\omega_\mathrm{d} t + \theta)\, \boldsymbol{f(\bar{\mathbf{x}}_t)}$$



Path Correlation

Efficient time sampling for

$$\cos(\omega_\mathrm{d} t + \theta)$$



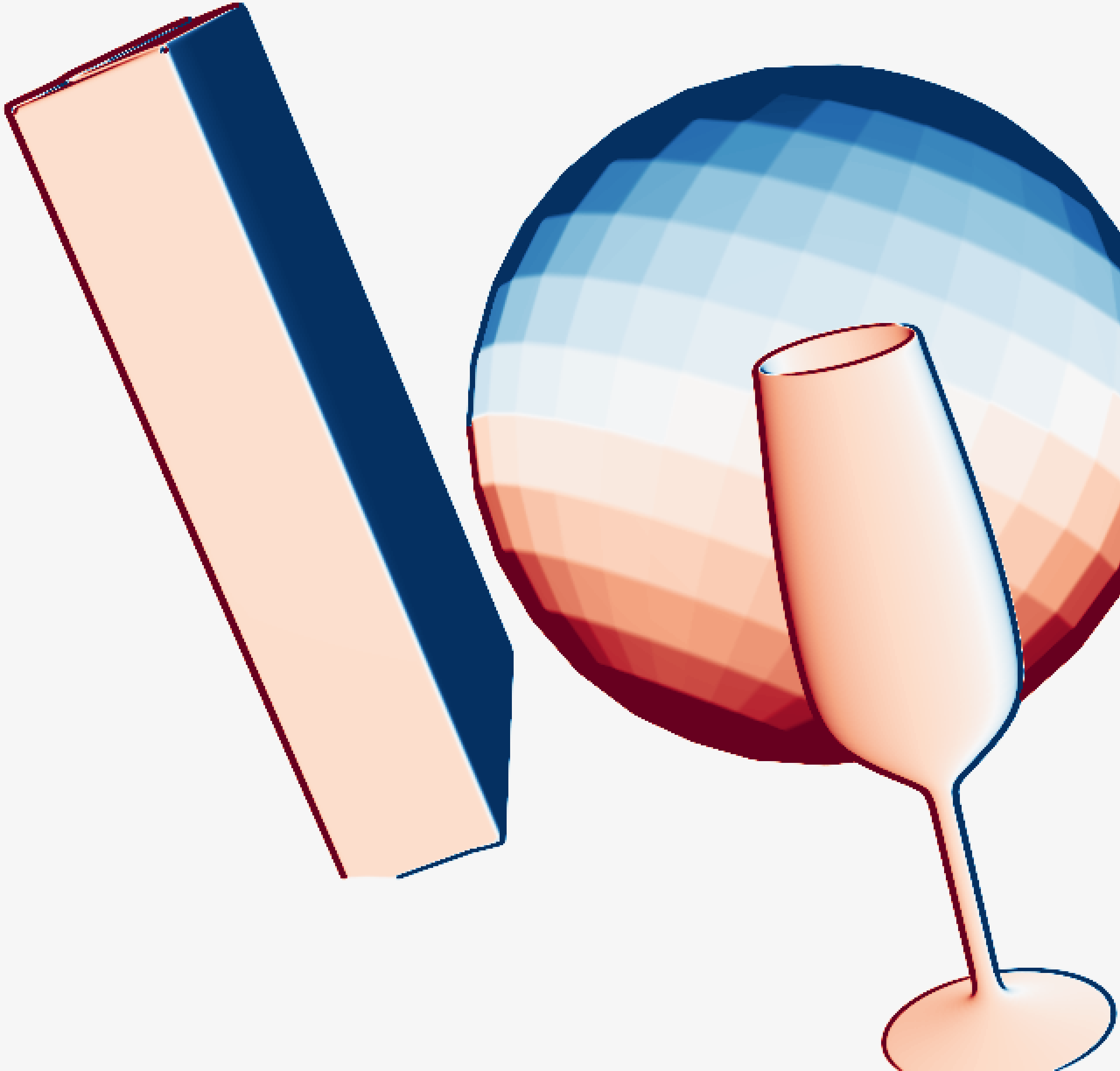
Antithetic Sampling

Unbiased Monte Carlo Estimate

Experiments & Result

Standard Rendering

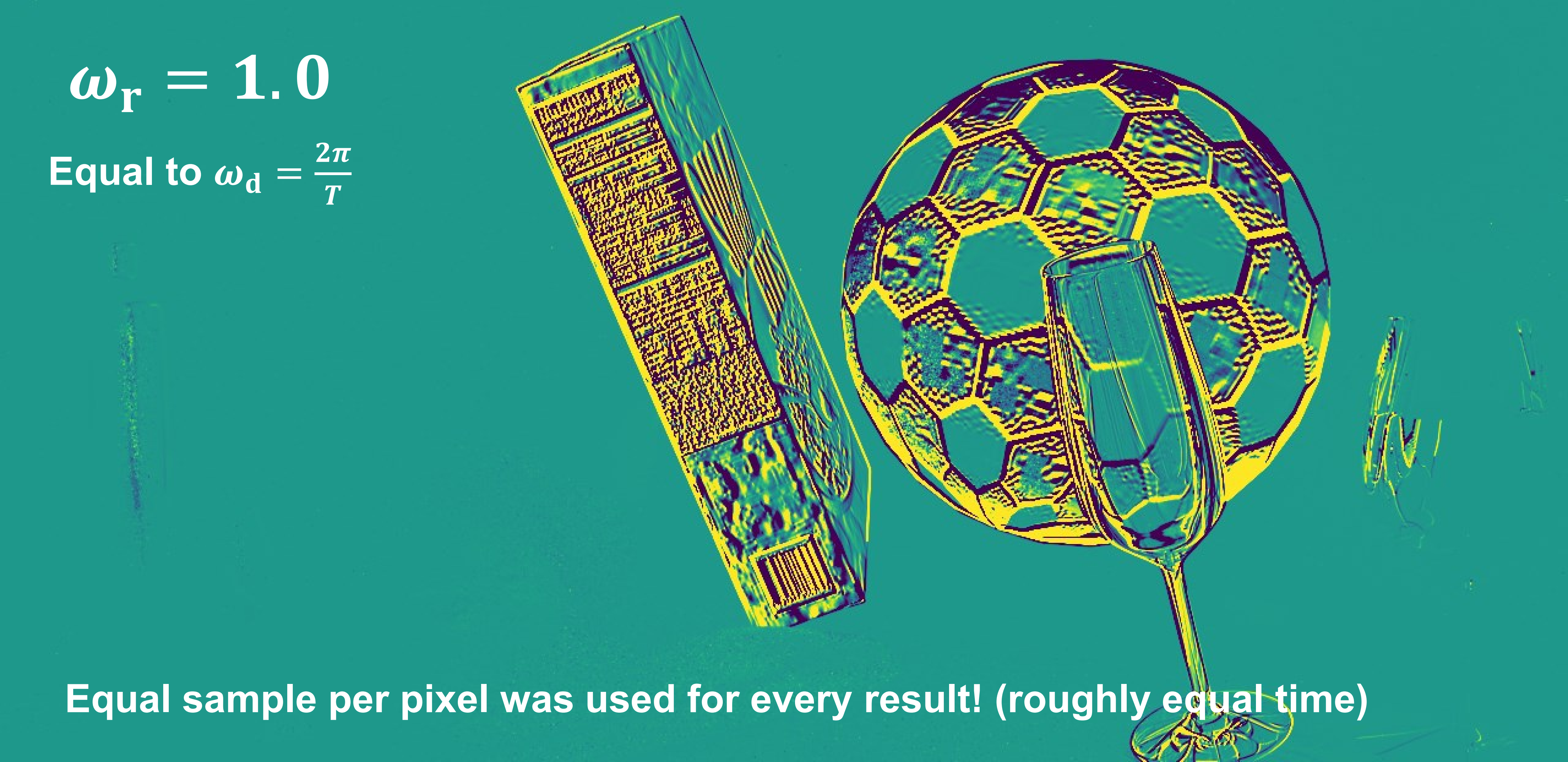
Per-Pixel Radial Velocity



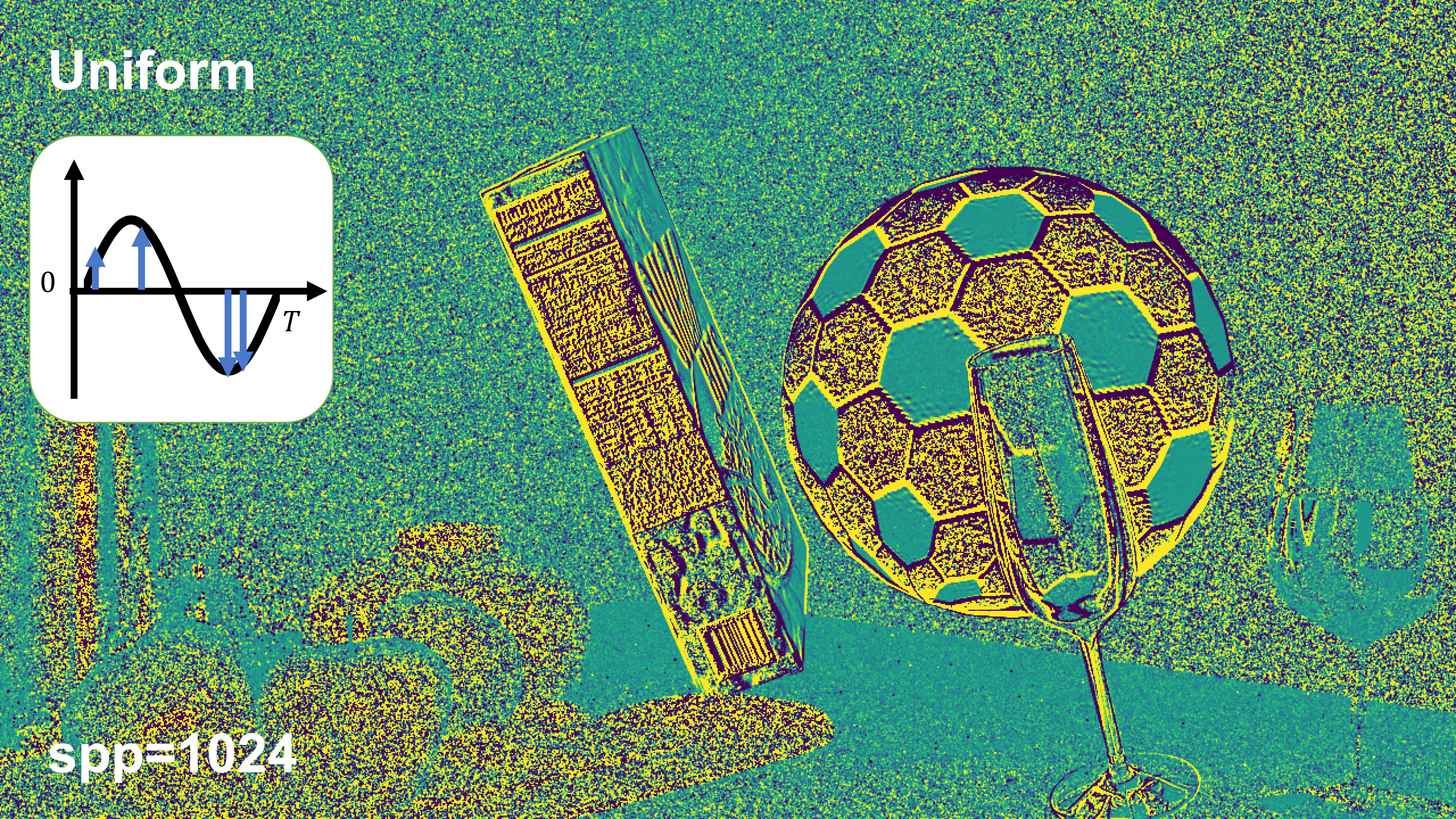
Reference D-ToF Image at

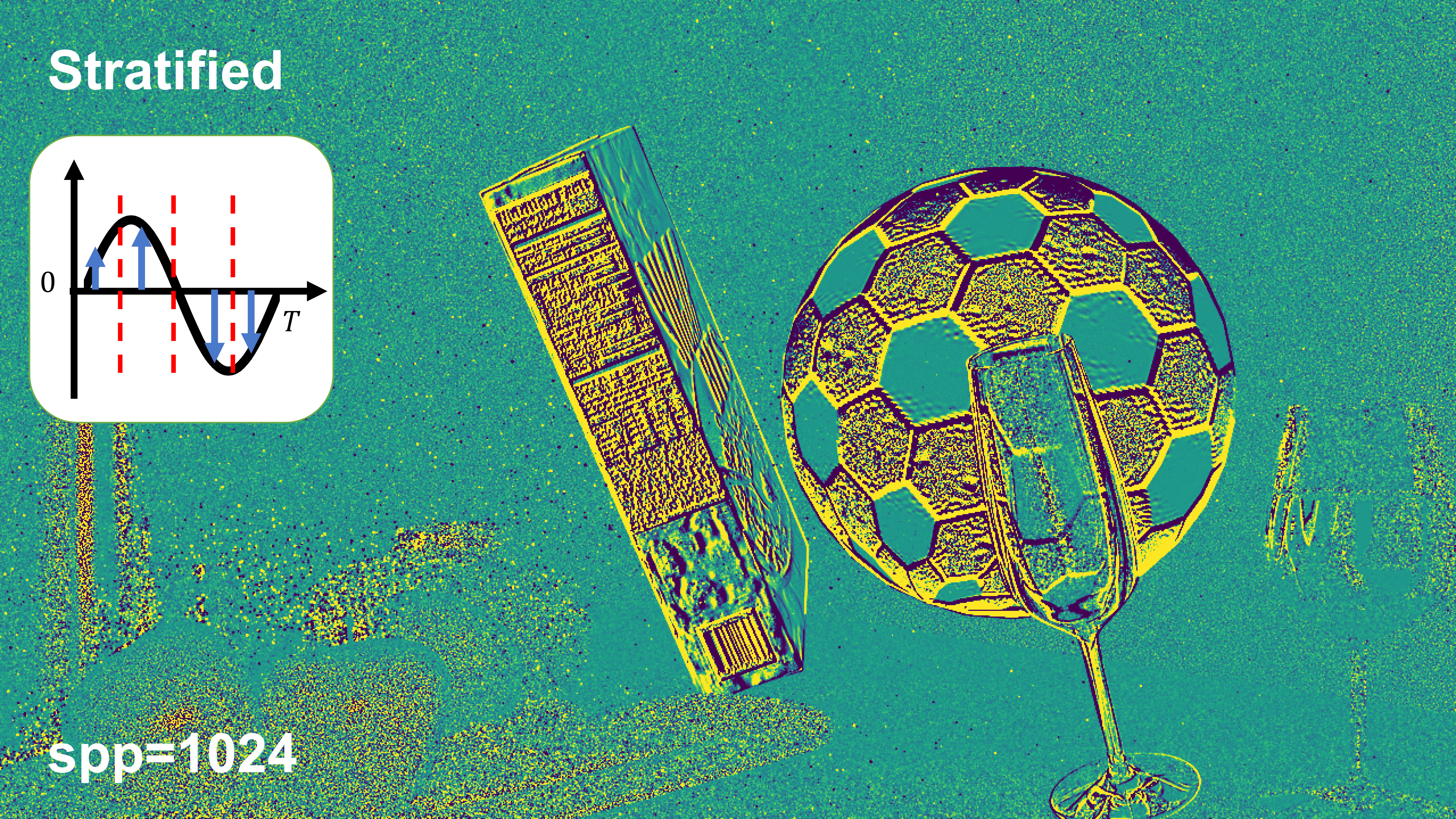
$$\boldsymbol{\omega_{\mathrm{r}} = 1.0}$$

Equal to $\omega_{\mathrm{d}} = \frac{2\pi}{T}$

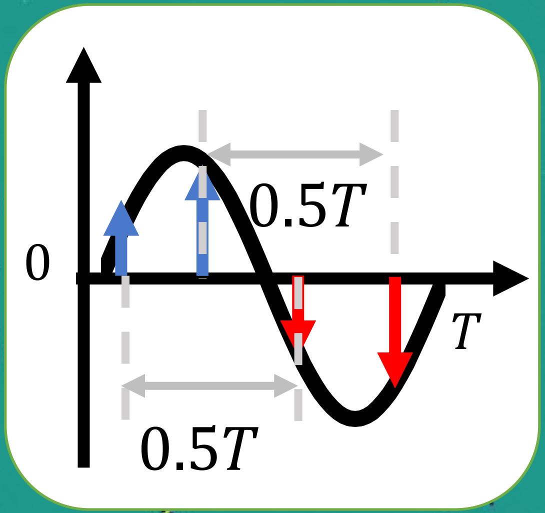


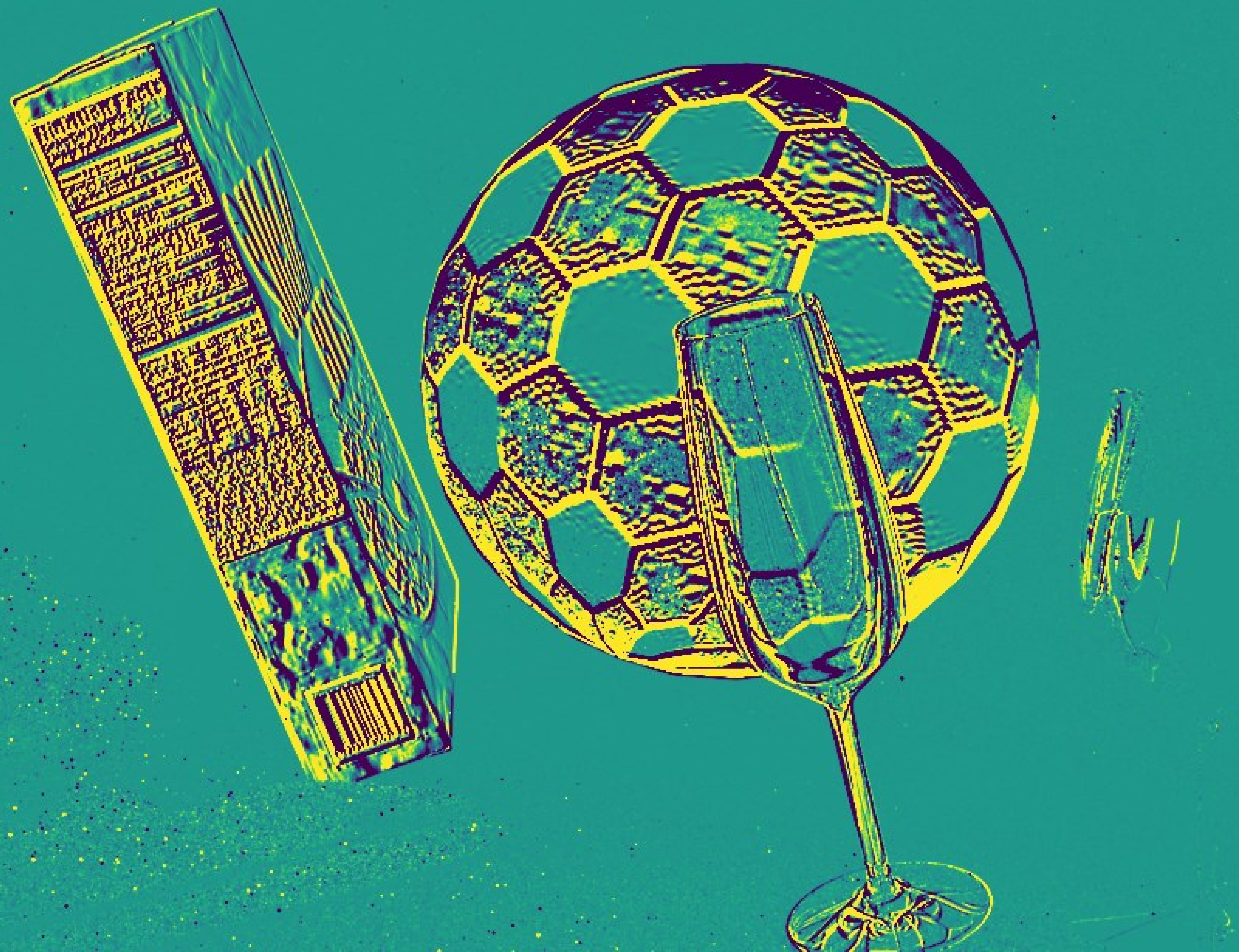
Equal sample per pixel was used for every result! (roughly equal time)


Uniform
0
T
spp=1024


Stratified
0
T
spp=1024

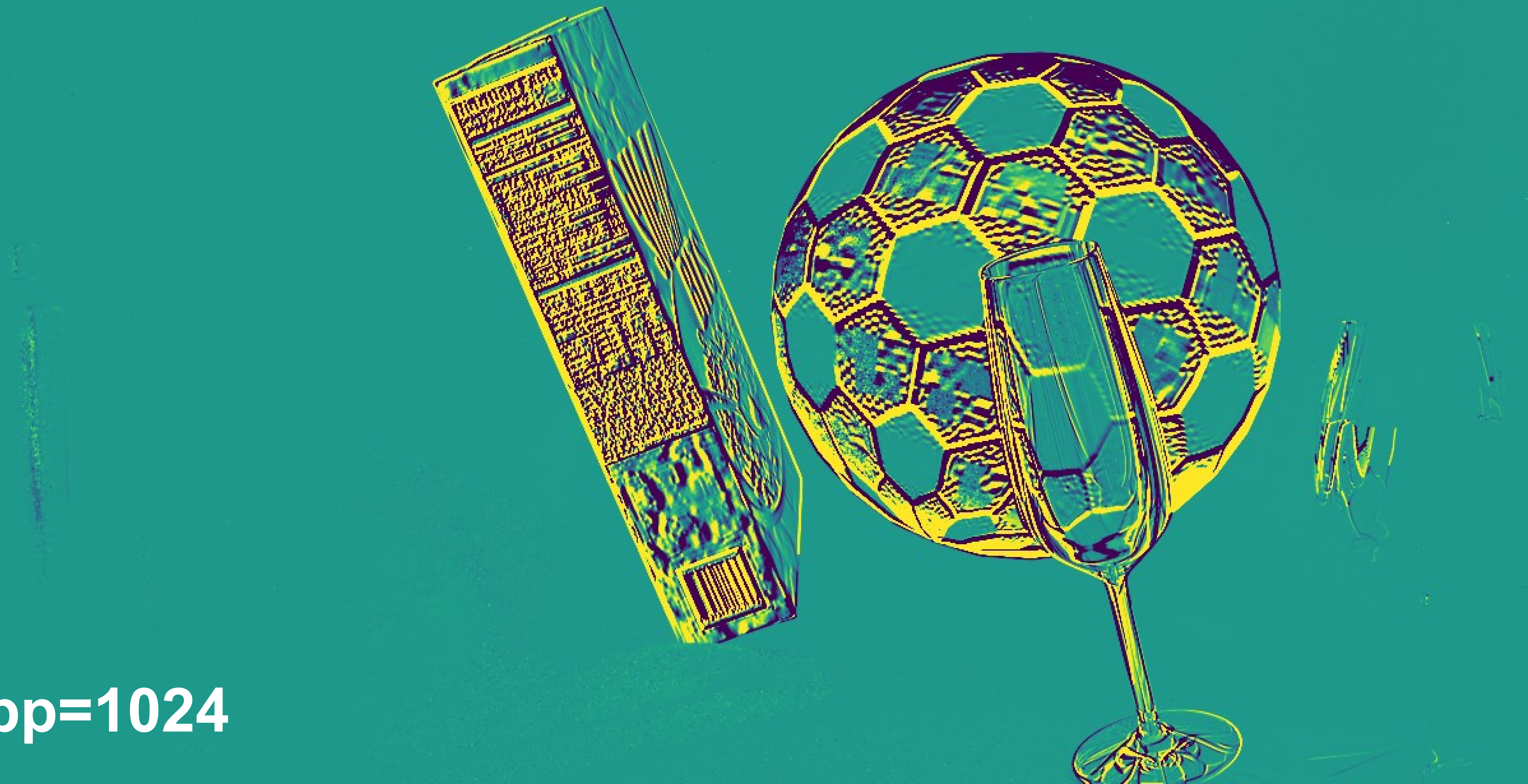
Antithetic (Proposed)



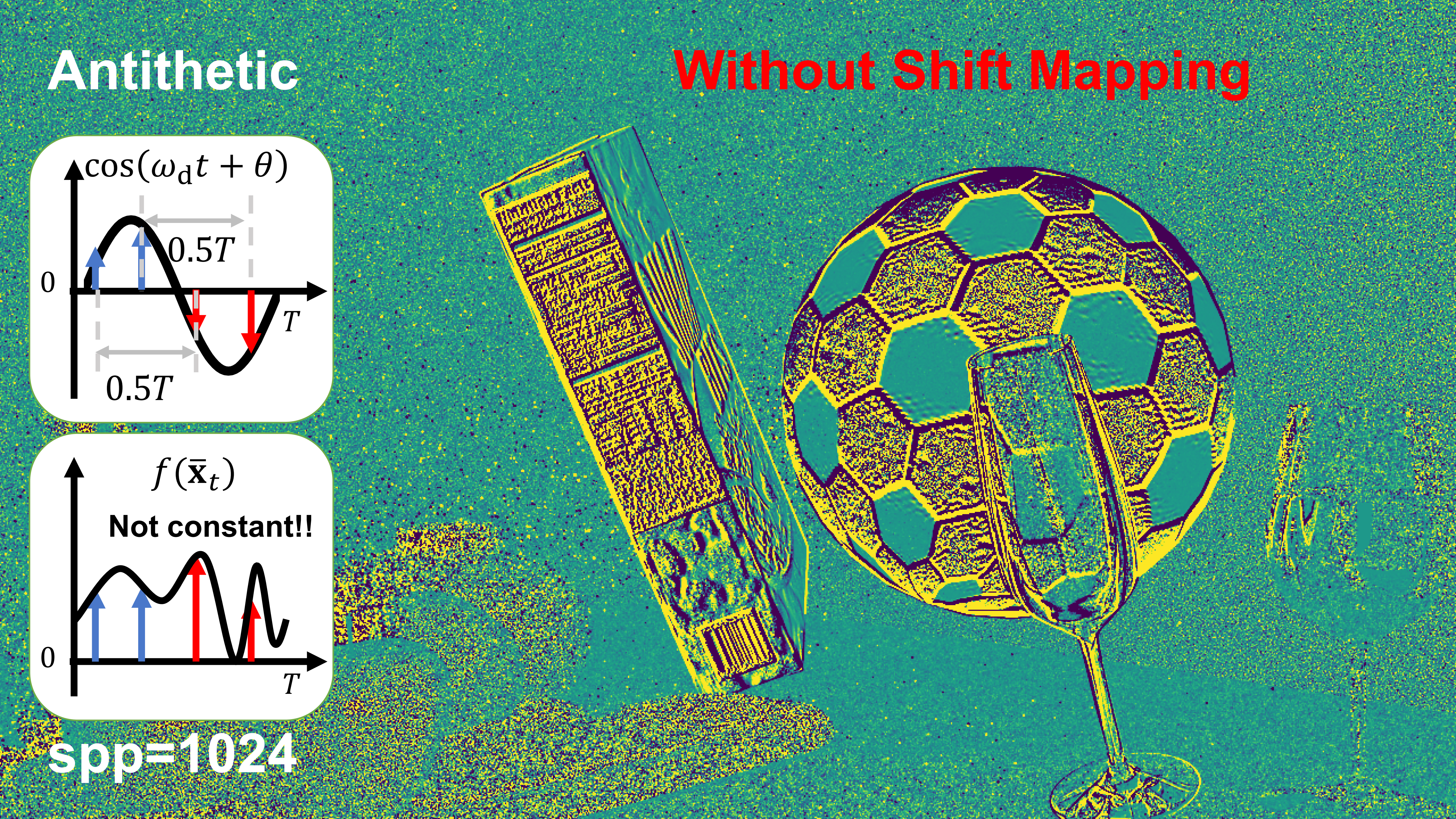


spp=1024

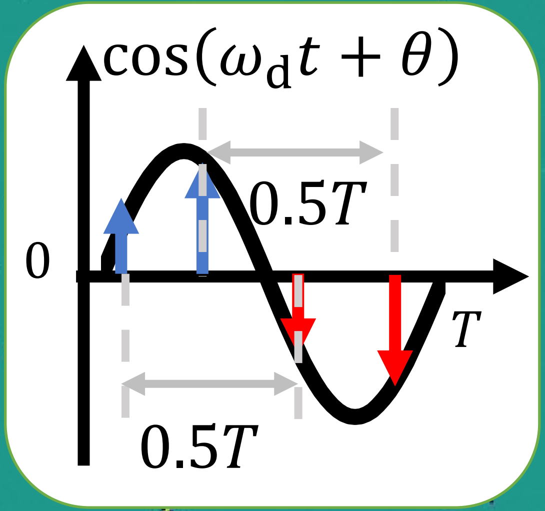
Reference

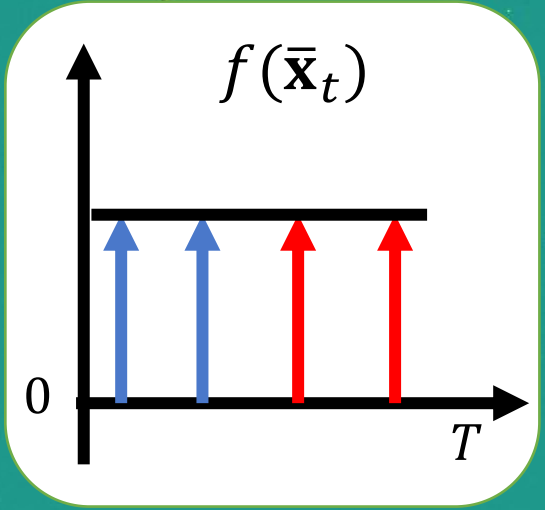


spp=1024


Antithetic
Without Shift Mapping
cos(ω_d t + θ)
0.5T
0
T
0.5T
f(x̄_t)
Not constant!!
0
T
spp=1024

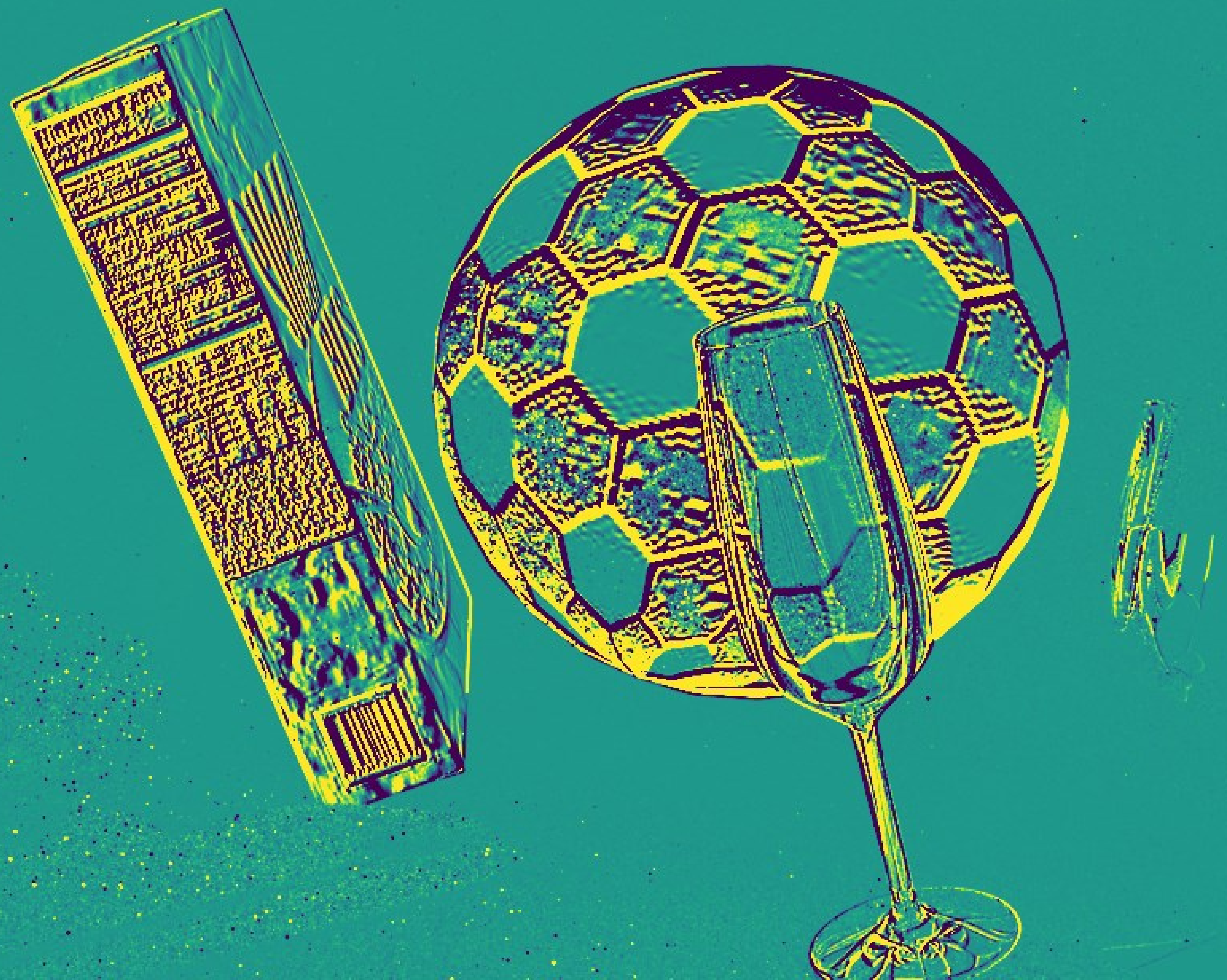
Antithetic

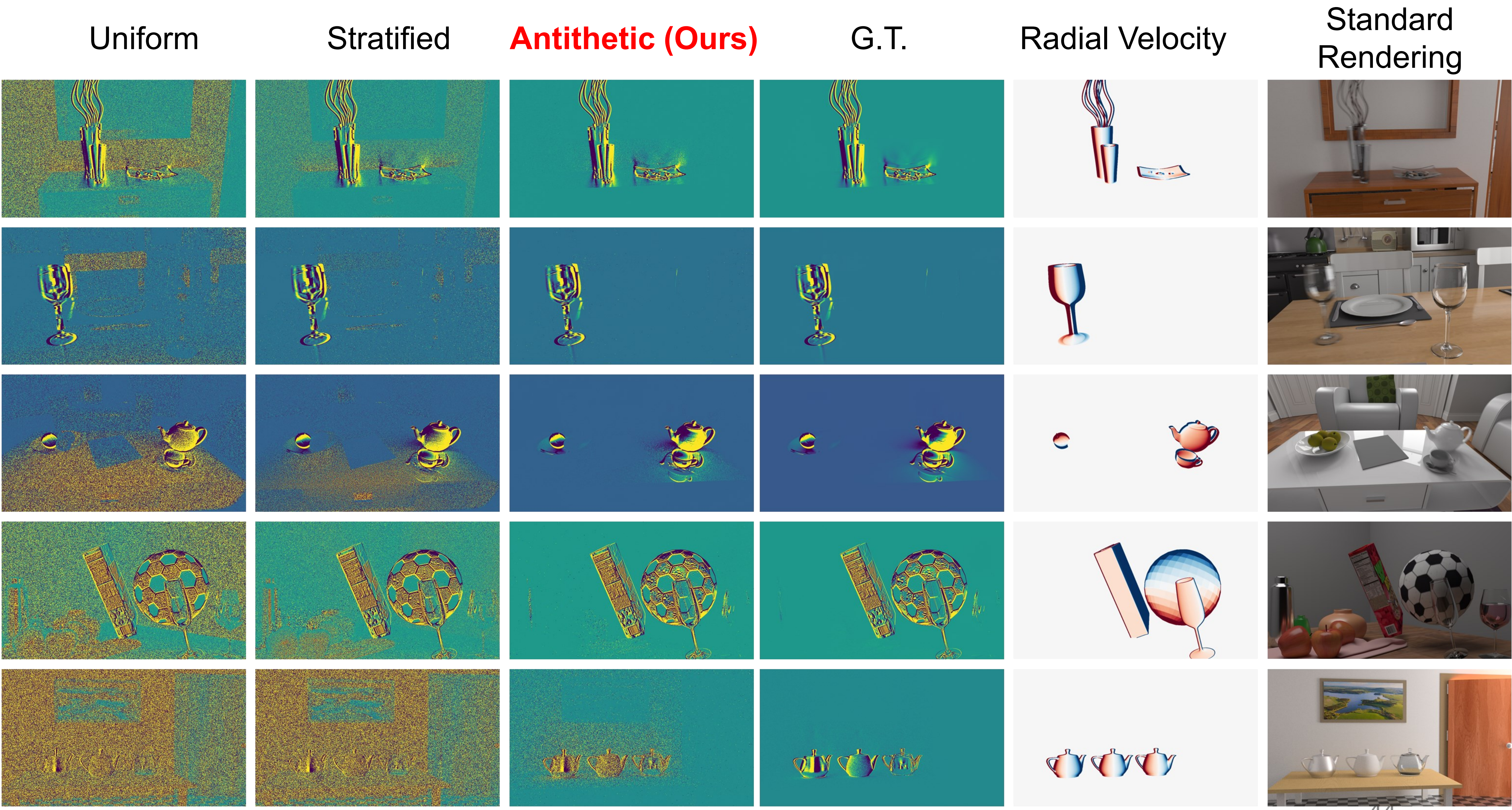




spp=1024

With Shift Mapping




Uniform
Stratified
Antithetic (Ours)
G.T.
Radial Velocity
Standard Rendering

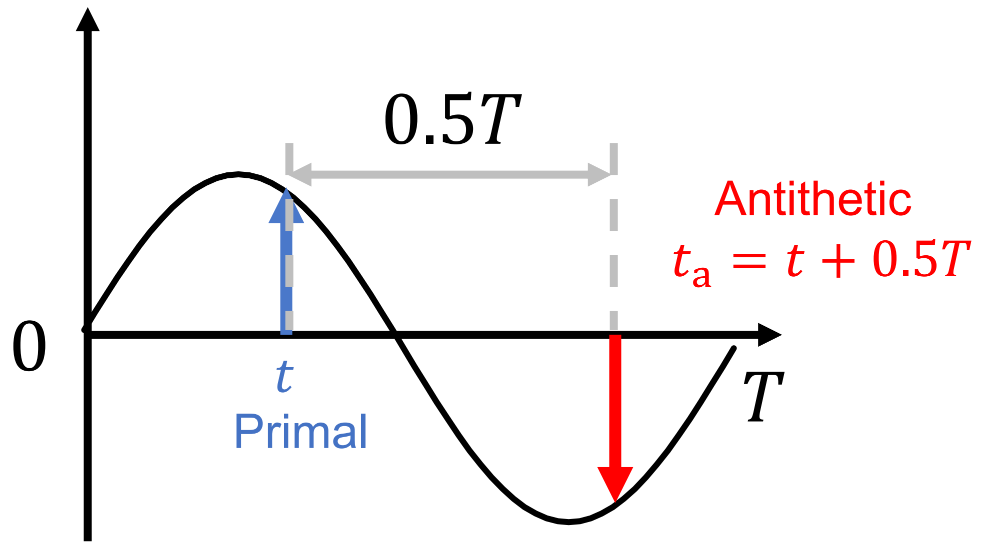
spp = 1024

Result 1: Comparison on Various Heterodyne Frequencies

[Heide 2015]

Perfect Heterodyne

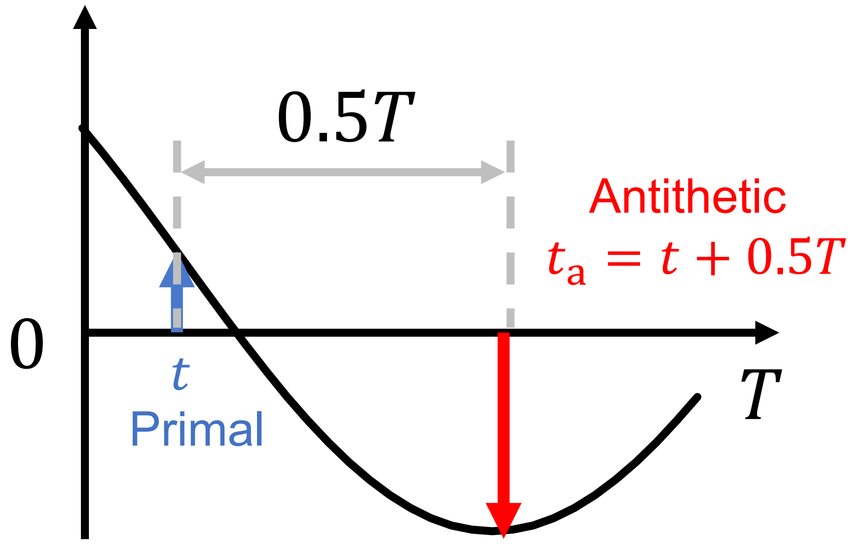
$\omega_r = 1$



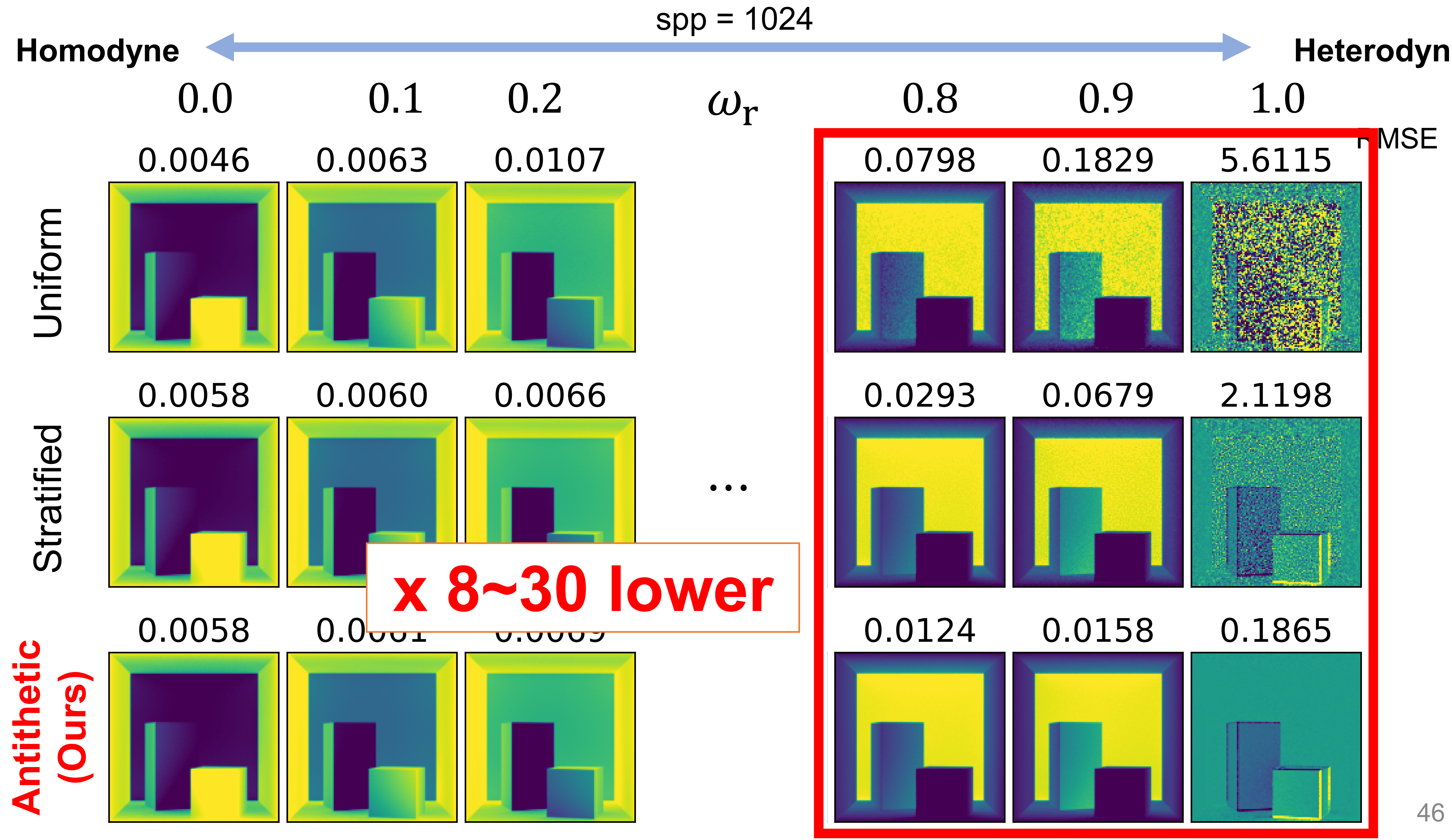
[Hu 2022]

Arbitrary Heterodyne

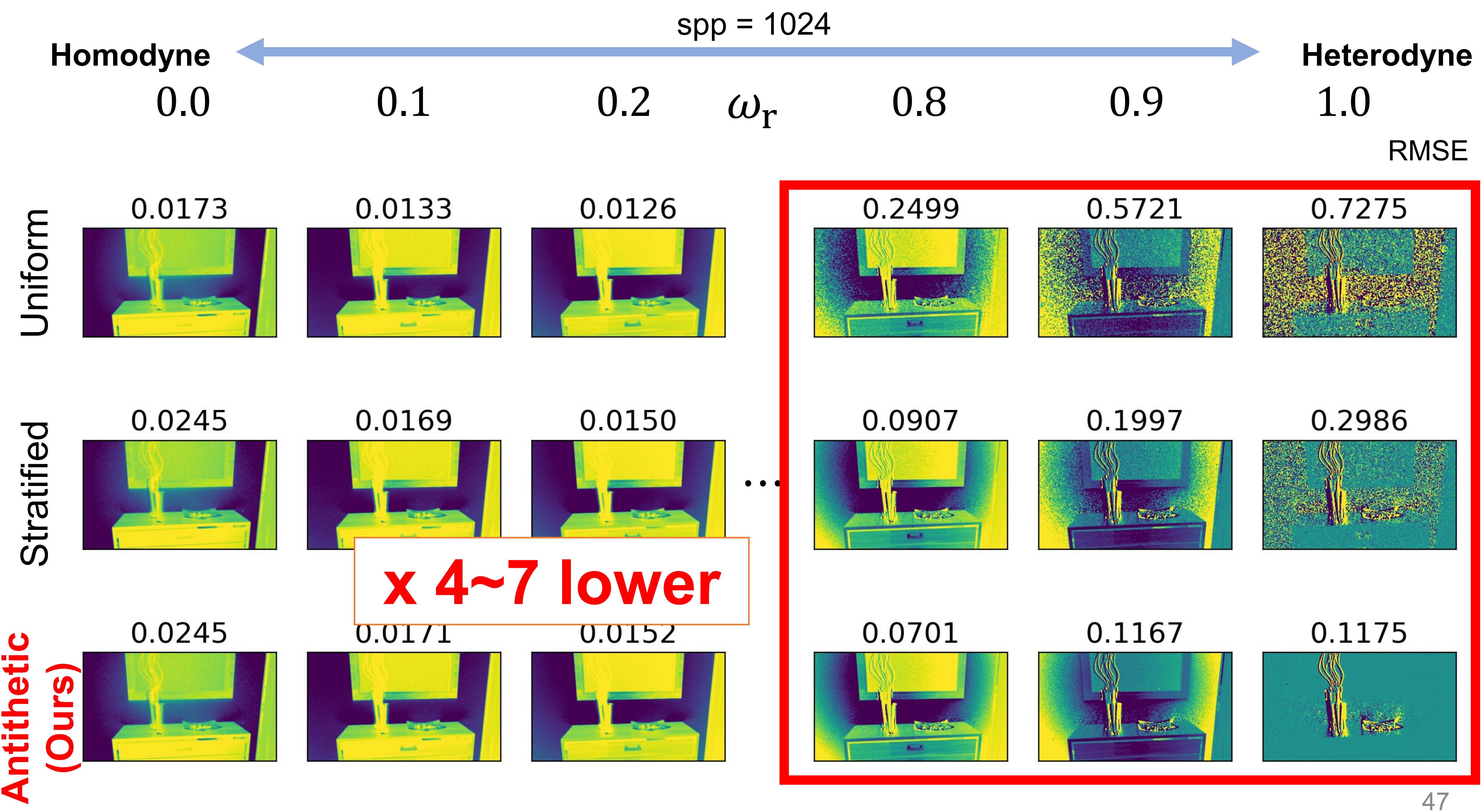
$0 < \omega_r < 1$



Does our method work well for other heterodyne frequencies?



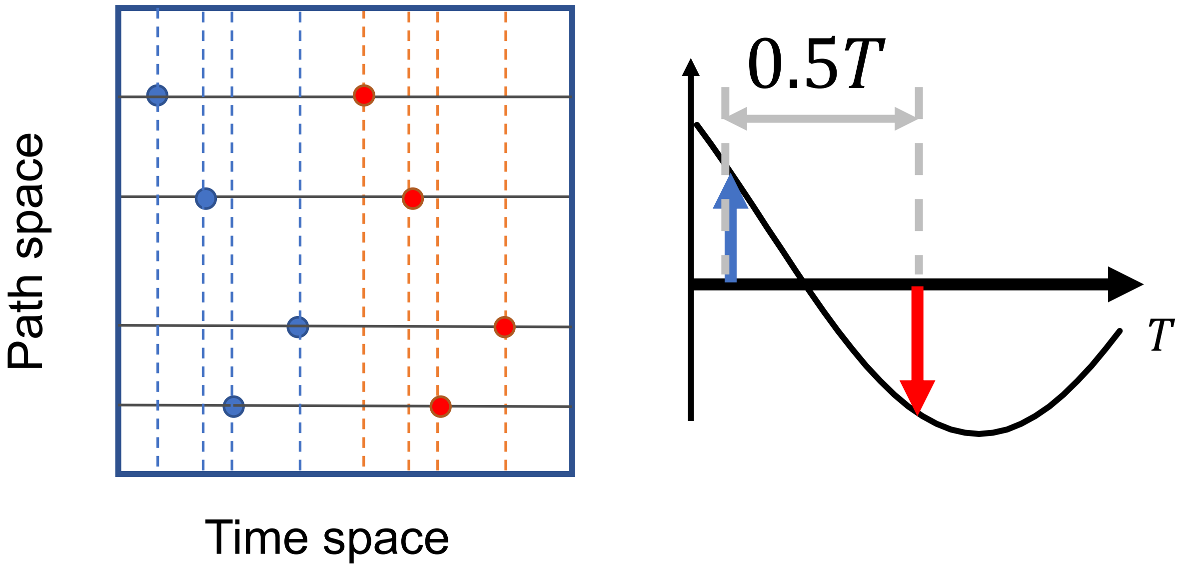
spp = 1024
Homodyne
Heterodyne
0.0
0.1
0.2
ω_r
0.8
0.9
1.0
RMSE
Uniform
0.0046
0.0063
0.0107
0.0798
0.1829
5.6115
Stratified
0.0058
0.0060
0.0066
0.0293
0.0679
2.1198
...
Antithetic (Ours)
0.0058
0.0124
0.0158
0.1865
x 8~30 lower



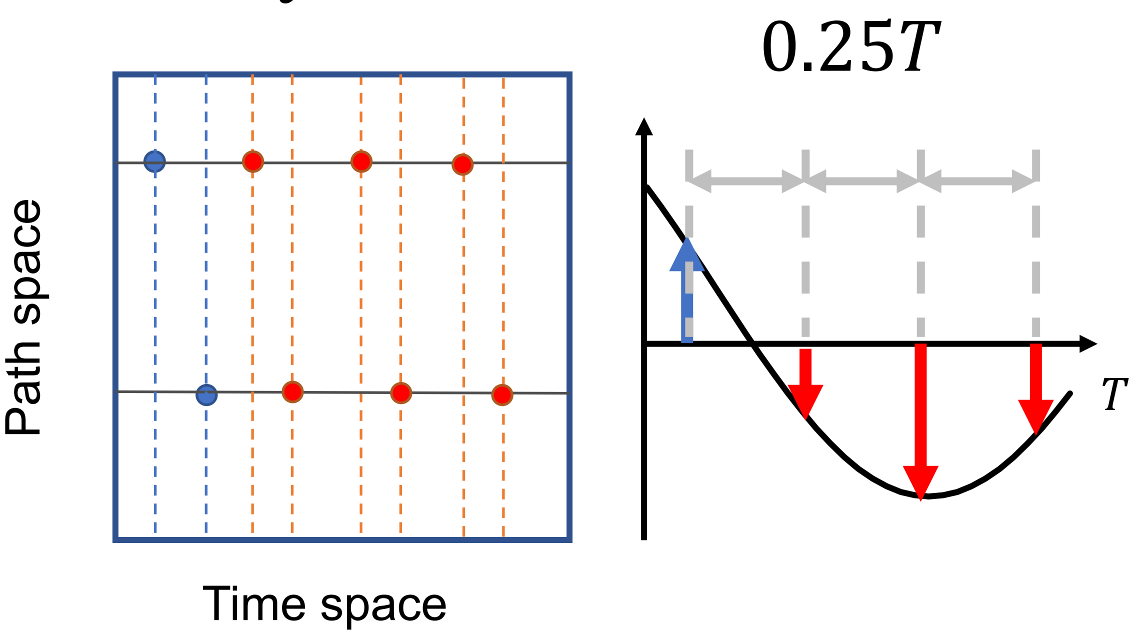
spp = 1024
Homodyne
Heterodyne
0.0
0.1
0.2
ω_r
0.8
0.9
1.0
RMSE
Uniform
0.0173
0.0133
0.0126
0.2499
0.5721
0.7275
Stratified
0.0245
0.0169
0.0150
0.0907
0.1997
0.2986
Antithetic (Ours)
0.0245
0.0171
0.0152
0.0701
0.1167
0.1175
x 4~7 lower

Result 2 : Comparison of Number of Time-Samples

$N_t = 2$

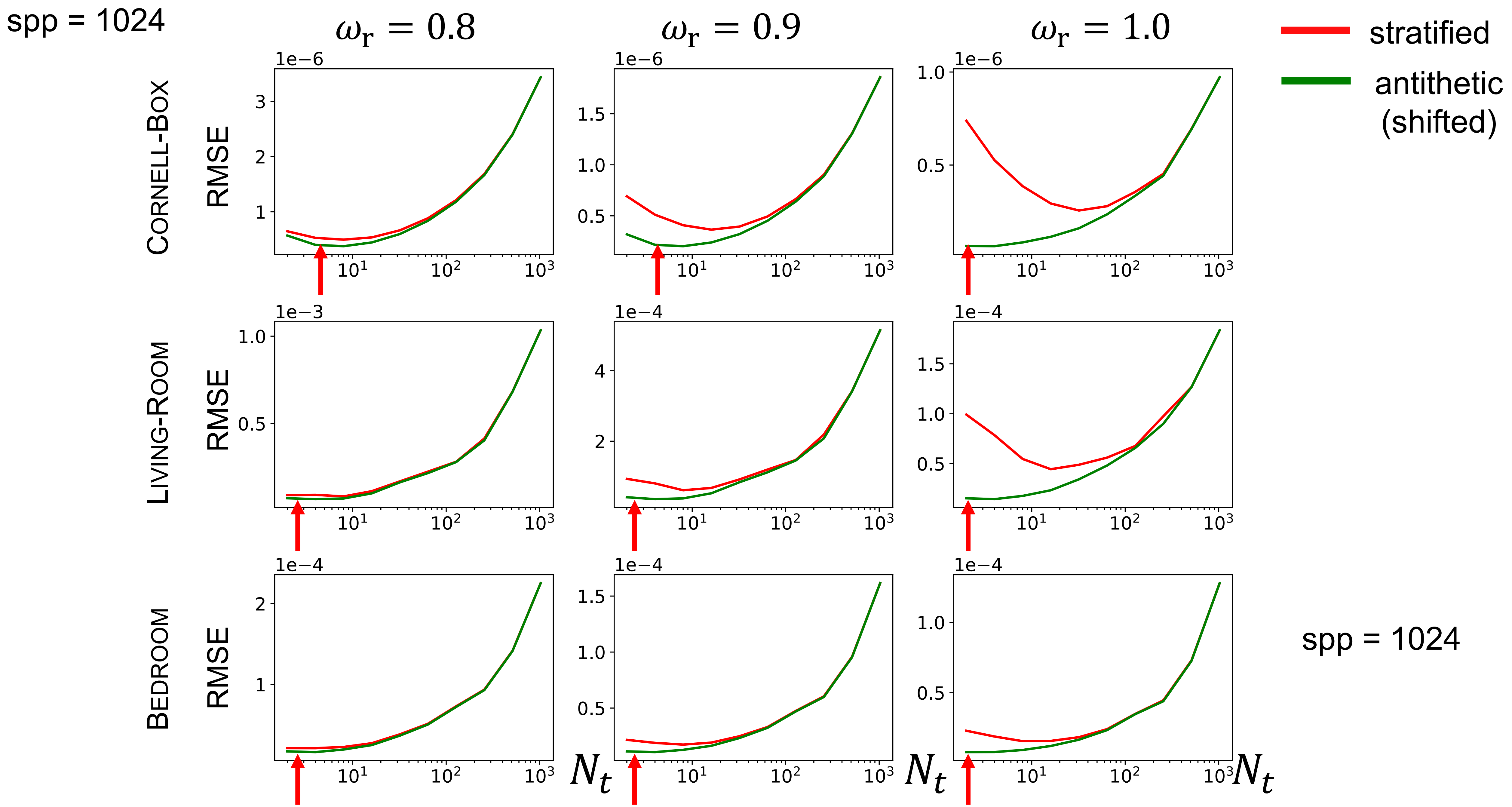


$N_t = 4$



Why only 2 time-samples? Can't we use more for better estimate for

$$\int \cos(\omega_{\mathrm{d}} t + \psi - \omega_g \|\bar{\mathrm{x}}_t\|)$$

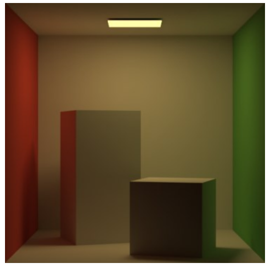


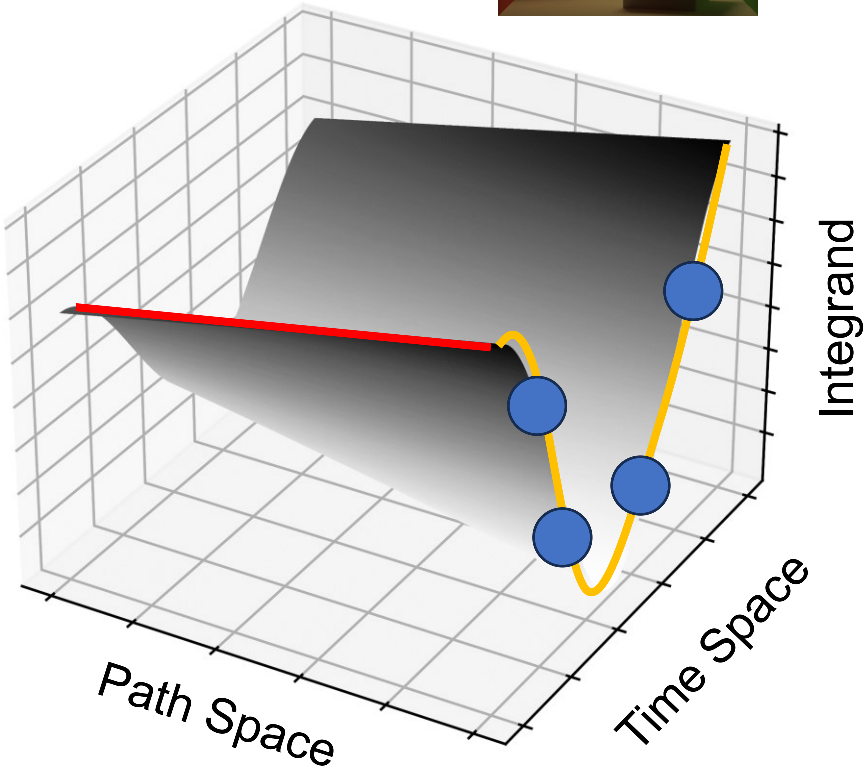
Antithetic sampling with only **one** sample ($N_t = 2$) works best in most of the cases

Result 2 : Comparison of Number of Time-Samples

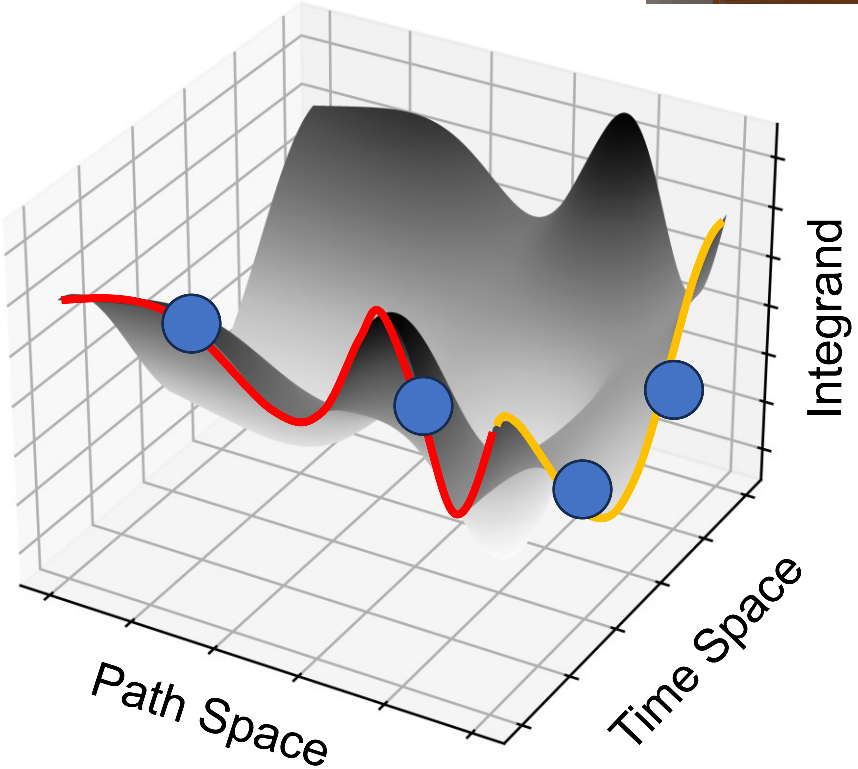
SIGGRAPH ASIA 2023 SYDNEY

Simple scene





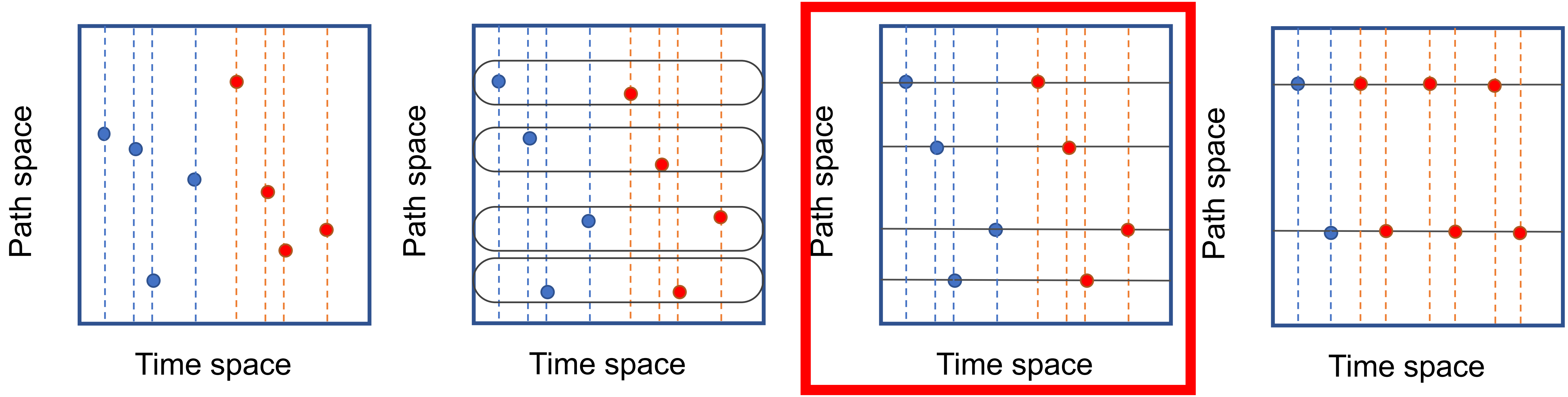
Complex scene



Can use more time samples

Need to consider path space!

Result 2 : Comparison of Path Correlation Strength



Variance in time space decreases ✓

Less correlation → More correlation

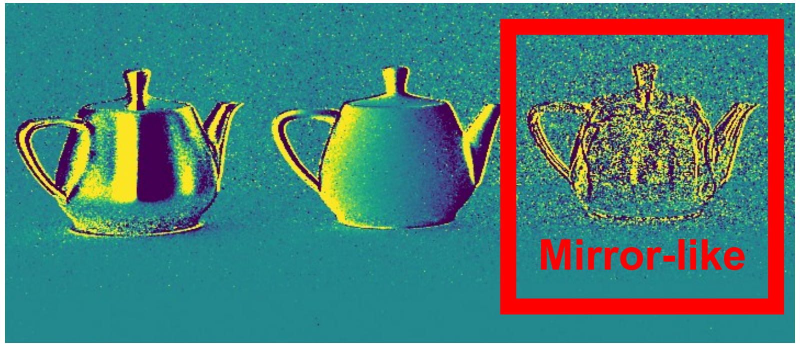
Variance in path space increases ✗

More result & discussion in main paper

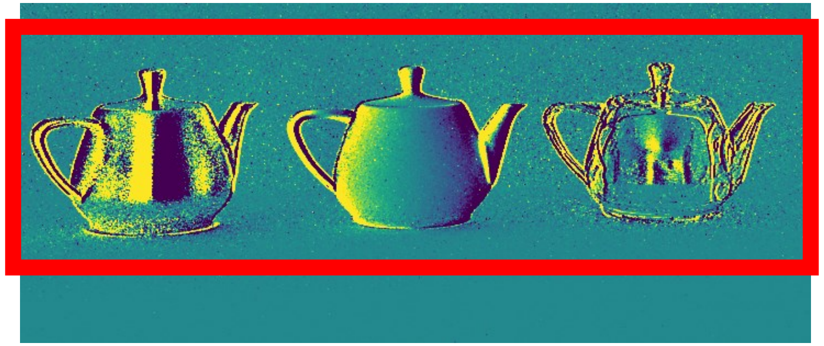
Result 3 : Comparison of Shift Mapping Methods

spp = 8192

Path Reconnection

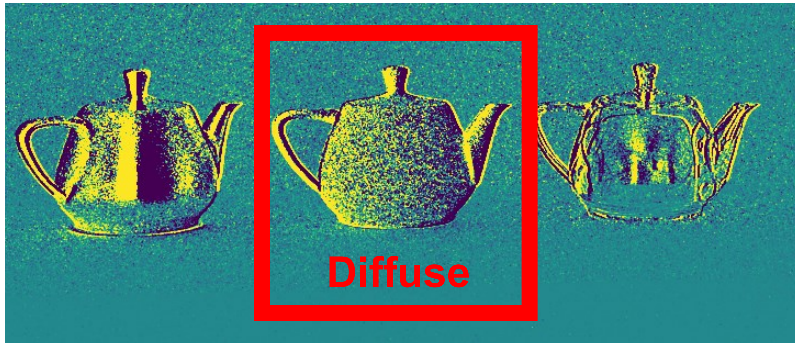


Adaptive

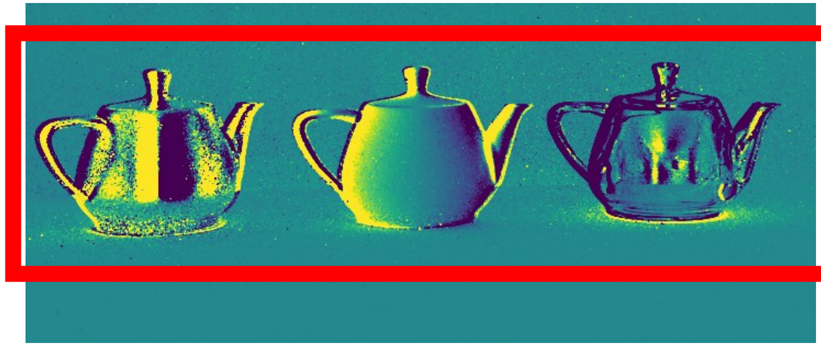


Reference

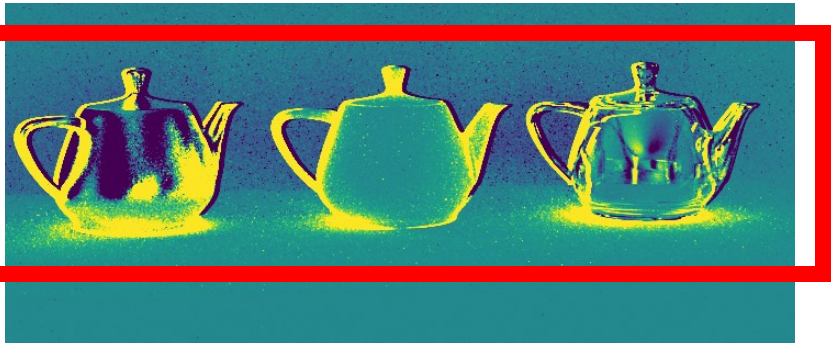
Random Replay



Adaptive (analytic)



Adaptive (analytic by [Heide 2015])



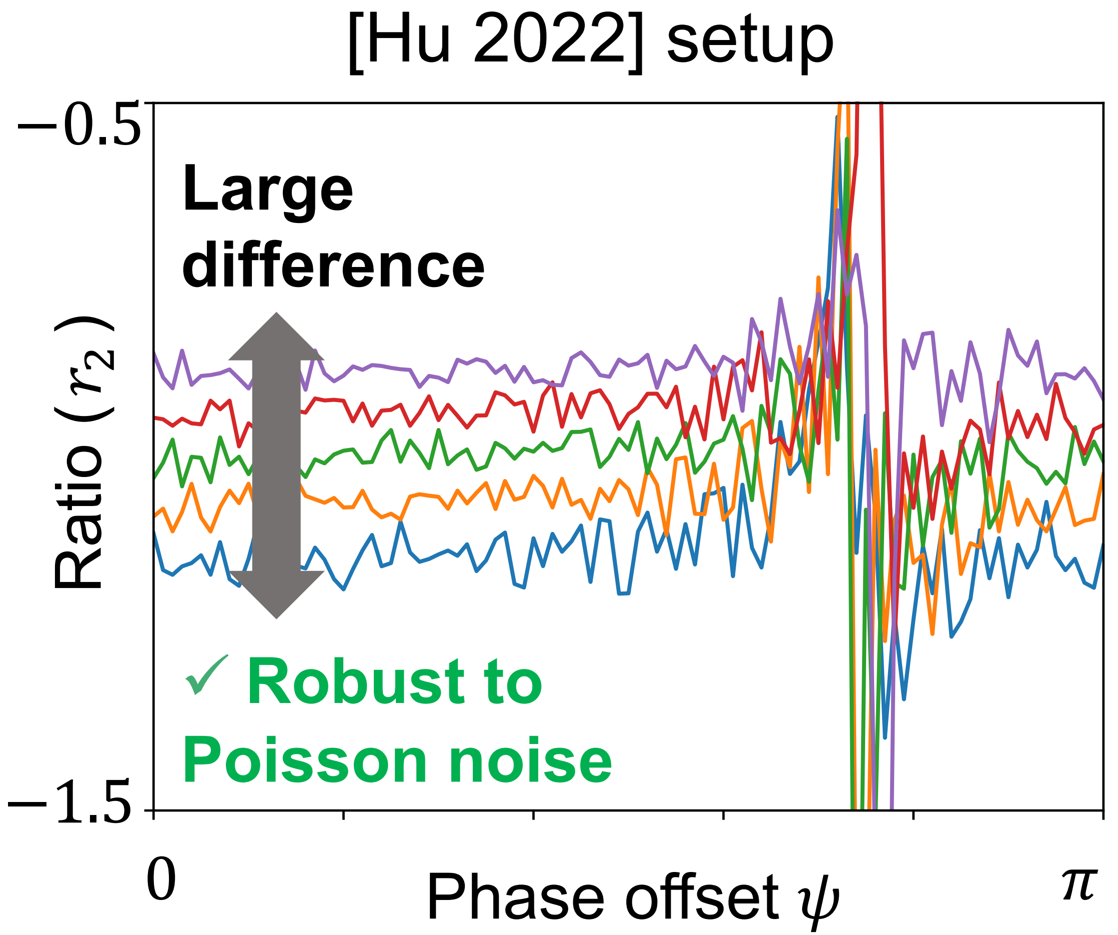
Considering Path Evolution (1st Taylor Approximation)

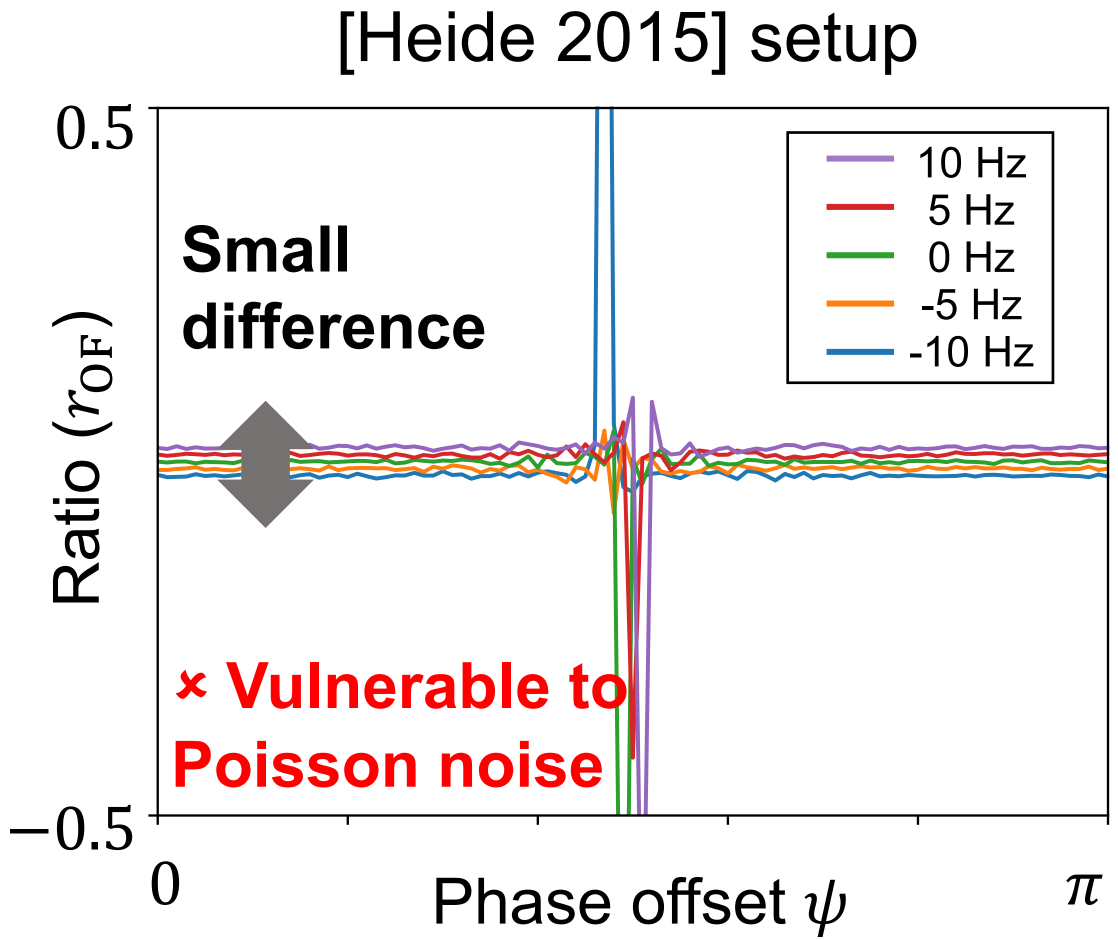
Not Considering Path Evolution (0th Taylor Approximation)

Path evolution : non-constant $f(\bar{\mathbf{x}}_t)$

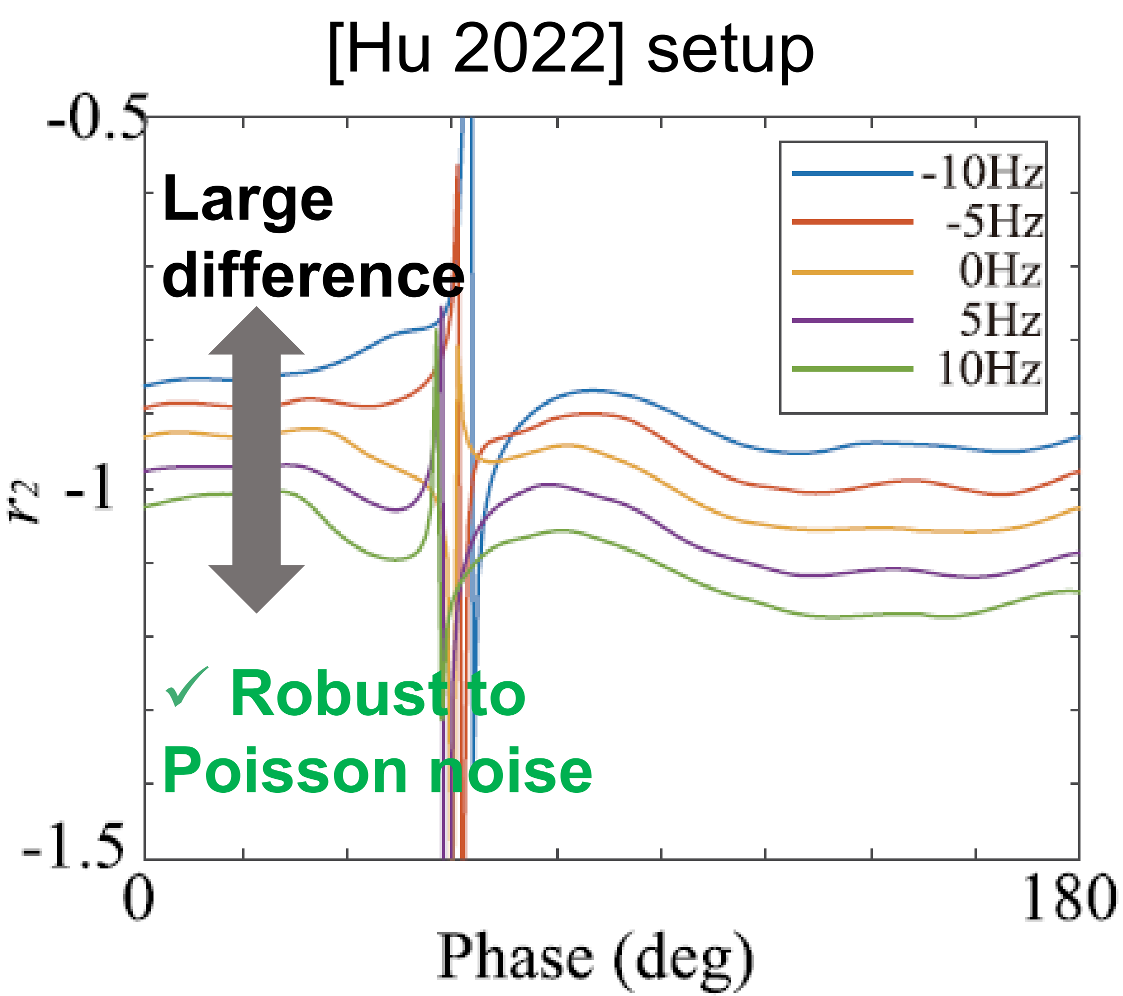
Applications of D-ToF Simulator

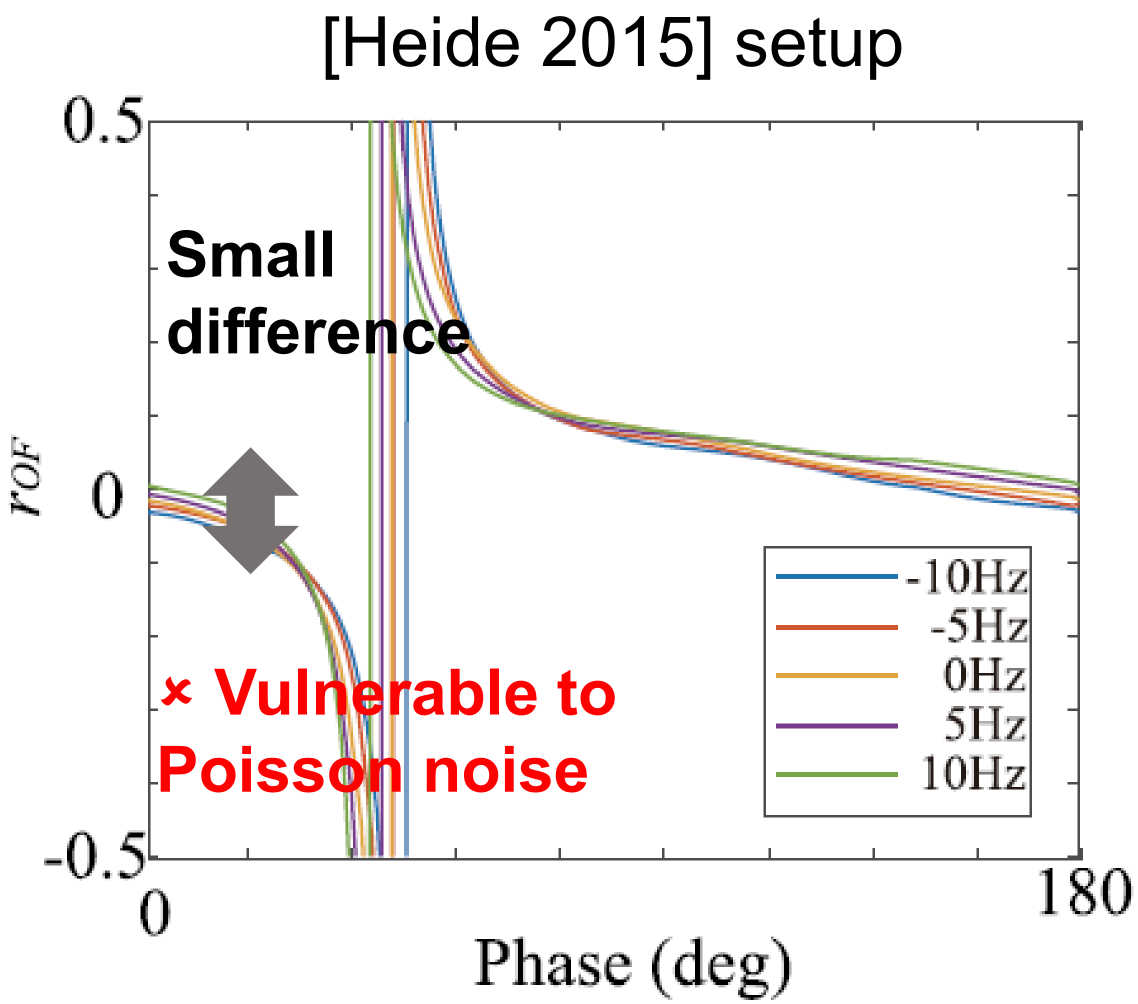
Reproducing D-ToF Paper Results (Simulation)





Reproducing D-ToF Paper Results (from [Hu 2022])

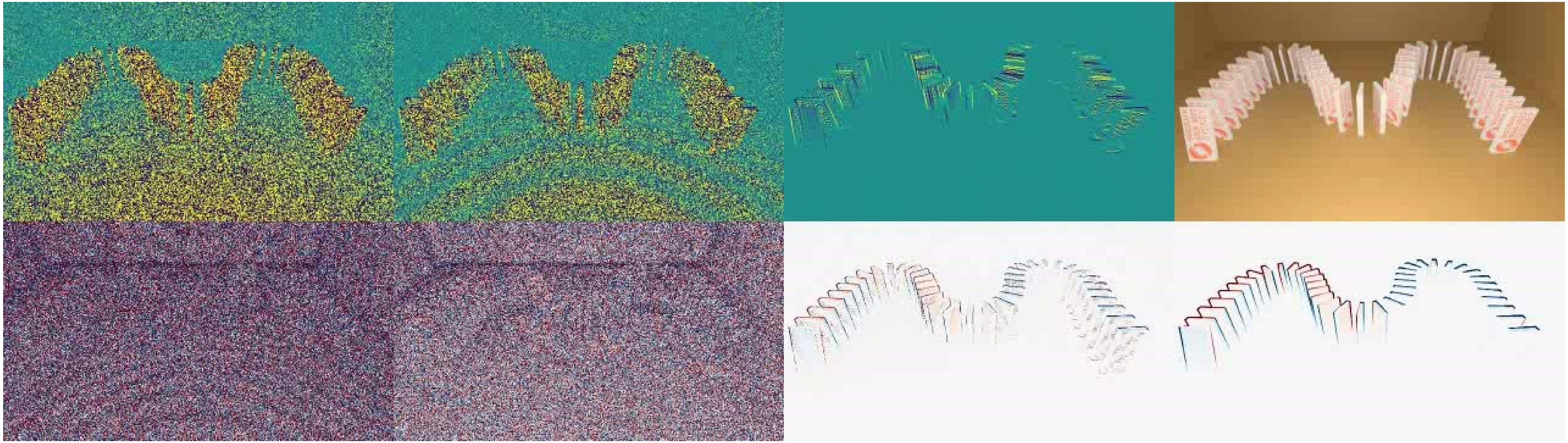




[Hu 2022], Fig 8.

More Results: SIGGRAPH DOMINO (SPP=4096)

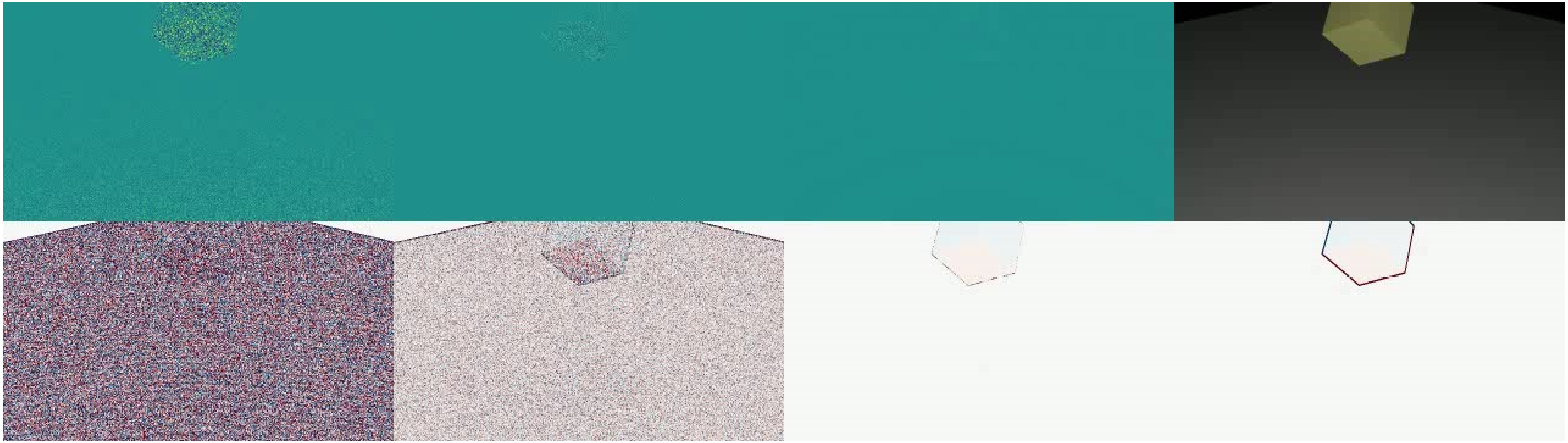
Uniform | Stratified | Proposed | Standard Rendering



Reconstructed Radial Velocity | GT Radial Velocity

More Results: FALLING-BOX (SPP=4096)

Uniform | Stratified | Proposed | Standard Rendering

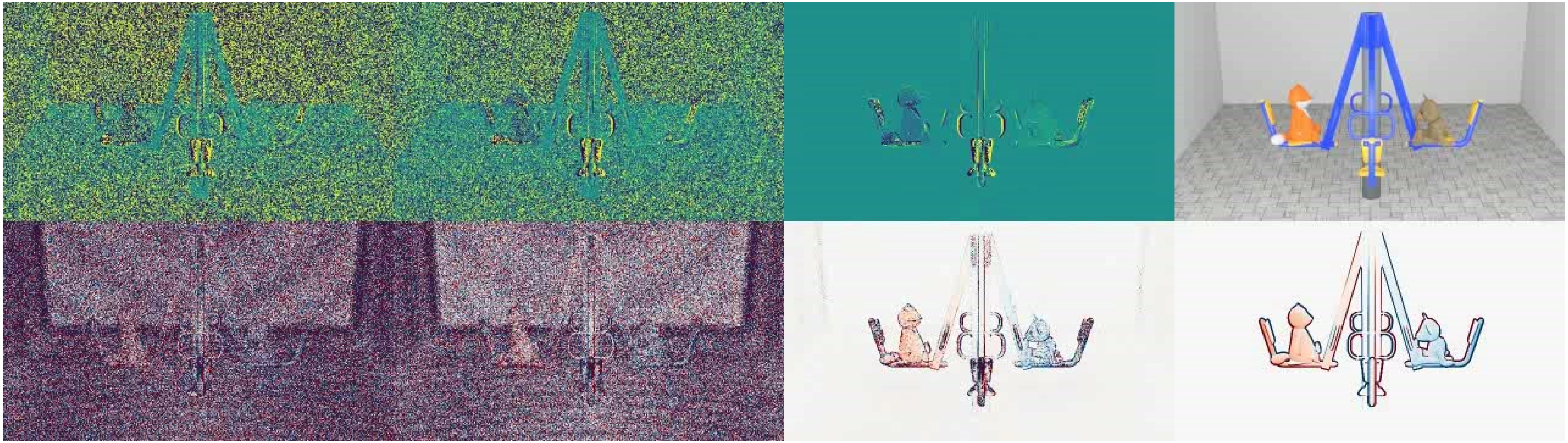


Reconstructed Radial Velocity

GT Radial Velocity

More Results: MERRYGOROUND (SPP=16384)

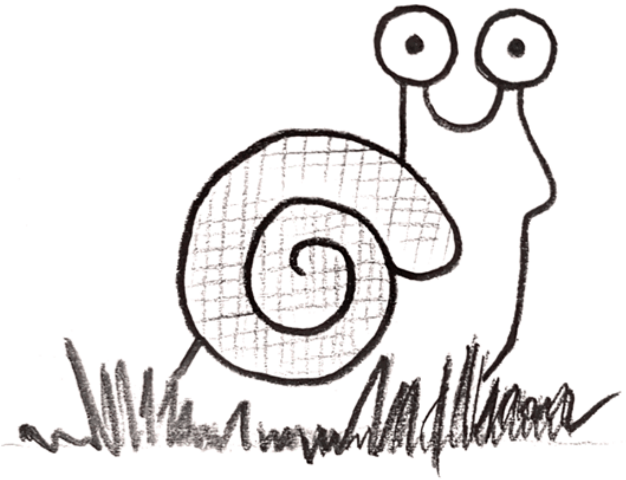
Uniform | Stratified | Proposed | Standard Rendering



Reconstructed Radial Velocity | GT Radial Velocity

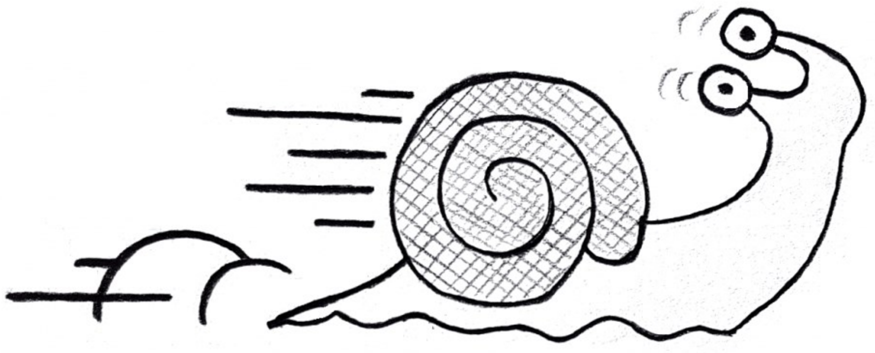
Conclusion

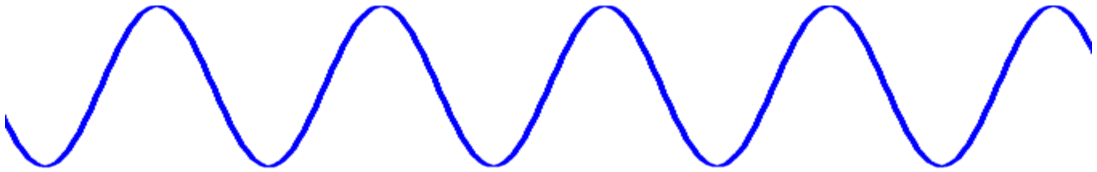
Static



Dynamic

Path Correlation





Amplitude Modulation

Antithetic Time Sampling

Thank you

Project Page : https://juhyeonkim95.github.io/project-pages/dopplertof/

Code for both

Mitsuba0.6 (CPU)

Mitsuba3 (CUDA)

are available!

Acknowledgments: This project was supported by NSF award 1844538, 1730147, 1900849, Sloan Research Fellowship and Burke research initiation award.